Mass of the Earth

Aim

I am going to find the mass of the Earth by doing an experiment

Underlying Physics

Newton's law of universal gravitation states

$F = \frac{GMm}{r^2}$ where F is the force of attraction between two objects of mass M and m and r is how far apart they are. G is the gravitational constant which is $6{\cdot}67\times10^{-11}$ N m^2 kg^{-2}.

If *m* is 1kg and *M* is the Earth, then the force acting on *m* at the surface of the Earth is

$F = \frac{6\cdot67\times10^{-11}\times M_{Earth}\times1}{r^2}$ where *r* is the radius of the Earth.

The radius of the Earth is $6{\cdot}4\times10^4$ m, so

$$F = \frac{6\cdot67\times10^{-11}\times M_{Earth}\times1}{(6\cdot4\times10^6)^2}$$

$$F = 1\cdot6\times10^{-24}\times M_{Earth}$$

The force (*F*) acting on 1 kg is its weight (*W*), and $W = mg = 1\times g$, where g is the acceleration due to gravity at the surface. So

$$g = 1\cdot6\times10^{-24}\times M_{Earth}$$

So if I measure *g*, I can find M_{Earth}.

I can measure g by rolling a trolley down a slope and measuring its acceleration.

If the slope is at an angle of θ to the horizontal then the acceleration of the trolley down the slope is given by

$$a = g\sin\theta$$

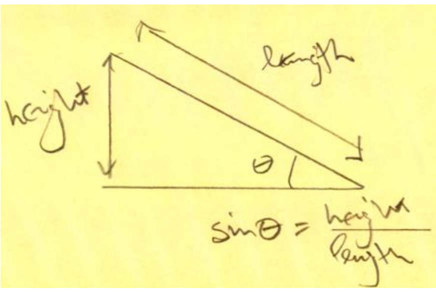
which is like

$$y = mx + c$$

Where y is *a*, m is *g*, x is *sinθ* and c is zero. So a graph of *a* vs. *sinθ* will give *g* as the gradient

Experiment to measure *g*.

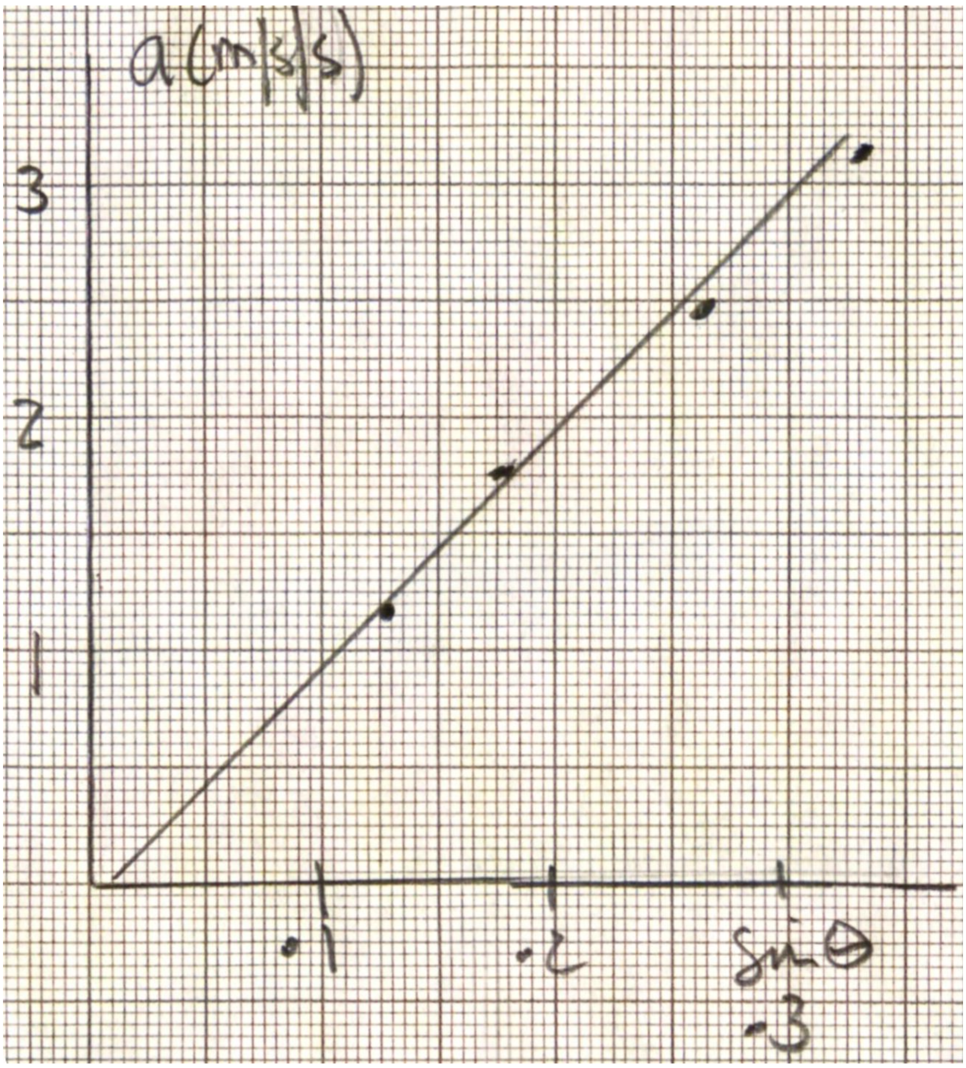
I measured the acceleration of the trolley with a notched card, a light gate and a microprocessor. I worked out the sine of the angle of the slope by measuring the length and height of the runway with a metre stick. I changed sinθ using a lab jack.



Table

Length(cm)	Height(cm)	sinθ	Acceleration (m/s/s)				
			First	Second	Third	Average	Random error
75.5	10	0.132	1.14	1.23	1.13	1.16	0.03
75.5	15	0.198	1.79	1.82	1.77	1.79	0.01
75.5	20	0.264	2.49	2.50	2.44	2.47	0.02
75.5	25	0.331	3.12	3.06	3.17	3.11	0.03

Graph



The gradient of the graph gives *g*

$$gradient = \frac{3-1}{0\cdot305 - 0\cdot105}$$
$$gradient = 10$$

So *g*=10 m/s/s

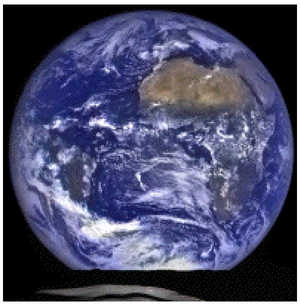
$g = 1\cdot6\times10^{-24}\times M_{Earth}$ so

$$M_{Earth} = \frac{10}{1\cdot6\times10^{-24}}$$
$$M_{Earth} = 6\cdot3\times10^{24} \text{ kg}$$

Data from Internet

I got this from NASA (1), and the mass of the Earth is $5{\cdot}9723\times10^{24}$ kg.

Earth Fact Sheet



Bulk parameters

Mass (10^{24} kg)	5.9723
Volume (10^{10} km^3)	108.321
Equatorial radius (km)	6378.137
Polar radius (km)	6356.752
Volumetric mean radius (km)	6371.008
Core radius (km)	3485
Ellipticity (Flattening)	0.00335
Mean density (kg/m^3)	5514
Surface gravity (m/s^2)	9.798
Surface acceleration (m/s^2)	9.780
Escape velocity (km/s)	11.186
GM (x 10^6 km^3/s^2)	0.39860
Bond albedo	0.306
Geometric albedo	0.434
V-band magnitude V(1,0)	-3.99
Solar irradiance (W/m^2)	1361.0

Analysis

The value I got and the value NASA got for the mass of the Earth are close to each other and about 6×10^{24} kg.

Evaluation

My internet source is very reliable because it is NASA, and they've measured the mass of the Earth very accurately.

My experiment worked well because I repeated my measurements and was very careful measuring the length and height of the runway.

If I was to do the experiment again, I would use a digital protractor, because it can measure angles more accurately than the metre stick method.

Reference

(1) https://nssdc.gsfc.nasa.gov/planetary/factsheet/earthfact.html

(Accessed January 2018)