Question 1

Candidate 1

QUESTION NUMBER		DO NOT WRITE IN THIS MARGIN

1.(a)	$M_{QR}\left(\dfrac{x_1+x_2}{2},\ \dfrac{y_1+y_2}{2}\right)$ $P(5,-1)$ gradient of $Q(-2,8)$ $p=3$ $R(13,3)$ $x_2\ y_2$
	
	 $\left(\dfrac{-2+13}{2},\ \dfrac{8+3}{2}\right)$ $\begin{array}{c} a\ \ b \\ P(5,-1) \\ x\ y \end{array}$
	
	 $\left(\dfrac{11}{2},\ \dfrac{11}{2}\right)$ $y-b=m(x-a)$
	
	 $(5.5,\ 5.5)$ $y-(-1)=3(x-5)$
	
	         $y+1=3x-15$
	 $M_{QR}=\dfrac{y_2-y_1}{x_2-x_1}$ $y=3x-15-1$
	         $y=3x-16$
	
	 $=\dfrac{3-8}{13-(-2)}$
	
	 $=\dfrac{-5}{15}$
	
	 $=-\dfrac{1}{3}$
	
	 $M_1\times M_2=-1$ $M_{\perp}=3$		

1.(b)	$M_{PR}=\dfrac{y_2-y_1}{x_2-x_1}$ $P(5,-1)$ $R(13,3)$ $x\ \ y$ $x_2\ y_2$ $=\dfrac{3-(-1)}{13-5}$ $\tan\theta=\dfrac{1}{2}$ $=\dfrac{4}{8}$ $\theta=\tan^{-1}\left(\dfrac{1}{2}\right)$ $=\dfrac{1}{2}$ $\theta=0.46$		

Candidate 2

QUESTION NUMBER

DO NOT WRITE IN THIS MARGIN

$Q\overset{x_1 \ y_1}{(-2,8)} \quad R\overset{x_2 \ y_2}{(13,3)}$

1.(a) $m_{QR} = \dfrac{3-8}{13-(-2)}$

$= \dfrac{-5}{15}$

$= -\dfrac{1}{3}$

$m\perp = 3$

$y - b = m(x - a)$

$y - (-1) = 3(x - 5)$

$\underset{-1}{y+1} = 3x - \underset{-1}{15}$

$y = 3x - 16$

1.(b) PR

$P\overset{x_1 \ y_1}{(5,-1)} \quad \overset{x_2 \ y_3}{(13,3)}$

R

P

13

$m = \tan\theta$
$m = \tan 45$
$\tan\dfrac{1}{3}$

$m = \dfrac{3 - (-1)}{13 - 5}$

$= \dfrac{4}{8}$

$= \dfrac{1}{2}$

$m = \tan\theta$

$= \sin\dfrac{1}{2}$

$= 30^{0}$

Candidate 3

QUESTION NUMBER

1.(a)

$x_c y_c$
$(-2,8)$

Q

R $(13,3)$
$x_2 \ y_2$

P $(5,-1)$
$a \ \ b$

DO NOT WRITE IN THIS MARGIN

$m_{QR} = \dfrac{y_2 - y_1}{x_2 - x_1}$

$M \ P_{Alt} = M1 \times m2 = 1$ for perp lines

$M_{palt} = 3$

$= \dfrac{3-8}{13-(-2)}$

$= \dfrac{-5}{15}$

$= \dfrac{-1}{3}$

$y - b = m(x-a)$

$y + 1 = 3(x-5)$

$y + 1 = 3x - 15$

$y = 3x - 16$

$3x - y - 16 = 0$

1.(b)

$m = \tan\theta$

$3 = \tan\theta$

$\tan^{-1}(3) = \theta$

$\theta = 71.57°$

Candidate 4

1.(b)

$m = \tan\theta$

$m = \tan\left(\frac{1}{2}\right)$

$m = 60°$

P R C

$P(5,-1) \ R(13,3)$
$x_1 y_1 \quad x_2 \ y_2$

$\dfrac{3 - (-1)}{13 - 5} = \dfrac{4}{8} , \dfrac{1}{2}$

Question 2

Candidate 5

2. | POINT | GRADIENT | EQUATION |

$$y = 2x^5 - 3x$$
$$\frac{dy}{dx} = 10x^4 - 3$$
$$y - b = m(x - a)$$

$$= 2(1)^5 - 3(1)$$
$$= 10(1)^4 - 3$$
$$y + 1 = 7(x - 1)$$

$$= 2 - 3$$
$$= 10 - 3$$
$$y + 1 = 7x - 7$$

$$= -1$$
$$= 7$$
$$y = 7x - 8$$

$$(1, -1)$$
$$m = 7$$

Candidate 6

2.

$$y = 2(1)^5 - 3(1)$$
$$= 2 - 3 = -1$$

$$y = 2x^5 - 3x$$

$$\frac{dy}{dx} = 10x^4 - 3$$

For SV, $\frac{dy}{dx} = 0$

$$10x^4 - 3 = 0$$

For SV, $\frac{dy}{dx} = 0$　　$10x^4 - 3 = 0$

$$y = 10x^4 - 3$$

$$y - b = m(x - a)$$
$$y + 1 = 7(x - 1)$$
$$y + 1 = 7x - 7$$
$$y = 7x - 6$$

$$y = 10x^4 - 3$$
$$y = 10(1)^4 - 3$$
$$y = 7$$
$$m = 7$$

Candidate 7

QUESTION NUMBER

2.

$y = 2x^5 - 3x$ $x = 1$

$\frac{dy}{dx} = \boxed{10}x^4 - 3$

$= 10(1)^4 - 3$

$= 10 - 3$

$y = 7$ $(1, 7)$

$y - b = m(x - a)$

$y - 7 = 10(x - 1)$

$y - 7 = 10x - 10$

$y = 10x - 3$

$-1 \mid 10 \quad \overset{x^3}{0} \quad \overset{x^2}{0} \quad \overset{x}{0} \quad -3$

DO NOT WRITE IN THIS MARGIN

Candidate 8

QUESTION NUMBER

2.

$y = 2x^5 - 3x$ $x = \underline{1}$

$\frac{dy}{dx} = 10x^4 - 3$

$= 10(1)^4 - 3$

$y = 7$ $(1, 7)$

$m = 2$

$m \cdot m = -1$

$m = \frac{-1}{2}$

$y - b = m(x - a)$

$y - 7 = \frac{-1}{2}(x - 1)$

$2y - 14 = -x + 1$

$2y = -x + 15$

DO NOT WRITE IN THIS MARGIN

Candidate 9

2.

$y = 2x^5 - 3x$

$y = 10x^4 - 3$

$y = 2x^3 - 3x$

$x = 1$, $y = 10(1)^4 - 3$

$= 10 - 3$

$y = 7$

Question 3

Candidate 10

3.

$$\int 7\cos\left(4x + \frac{\pi}{3}\right)$$

$$= \frac{1}{7}\sin\left(4x + \frac{\pi}{3}\right) \, dx + C$$

Candidate 11

3.
$$\int 7\cos\left(4x + \frac{\pi}{3}\right) dx.$$

$$= 7\sin\left(4x + \frac{\pi}{3}\right) + c.$$

$$= 7\sin\left(4x + \frac{\pi}{3}\right) + c \quad \div 4.$$

$$= \frac{7}{4}\sin\left(4x + \frac{\pi}{3}\right) + c.$$

sin
cos
-sin
-cos

Question 4

Candidate 12

4.

$y = f(x)$

Candidate 13

4.
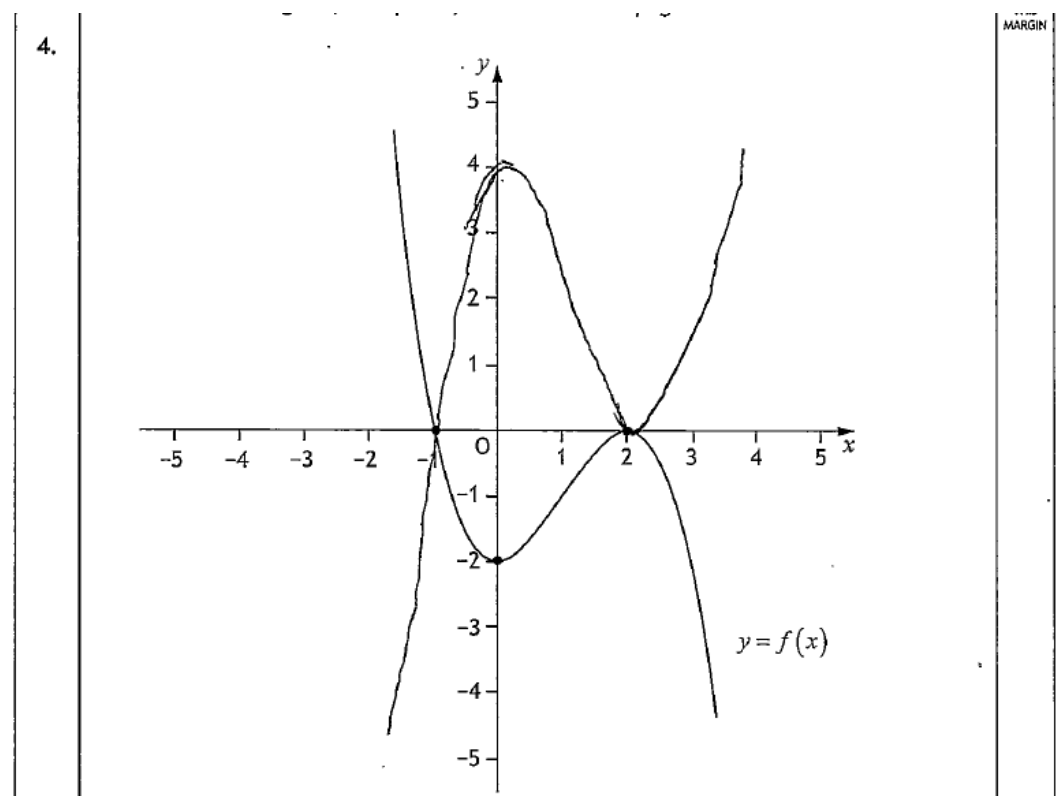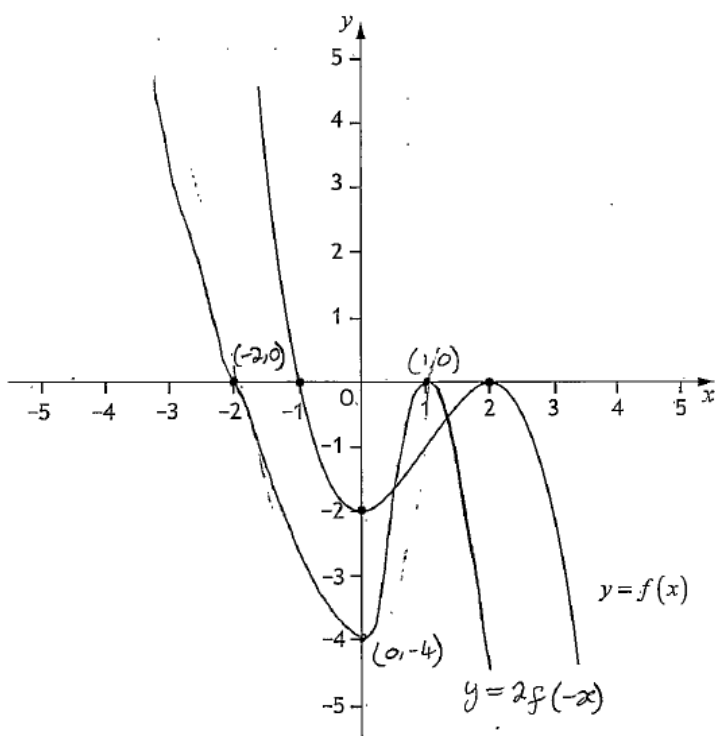
$y = 2 f(\boxed{-x})$

Stretch by 2
y-axis

→ flip on x-axis

$(-1, 0) \rightarrow (1, 0)$

$(2, 0) \rightarrow (-2, 0)$

$(0, -2) \rightarrow (0, -2)$
↓
$(0, -4)$

Candidate 14

4.

$y = 2f(-x)$

$y = f(x)$

$f(x)$	$2f(x)$	$2f(-x)$
$(-1, 0)$	$(-1, 0)$	$(1, 0)$
$(0, -2)$	$(0, -4)$	$(0, -4)$
$(2, 0)$	$(2, 0)$	$(-2, 0)$
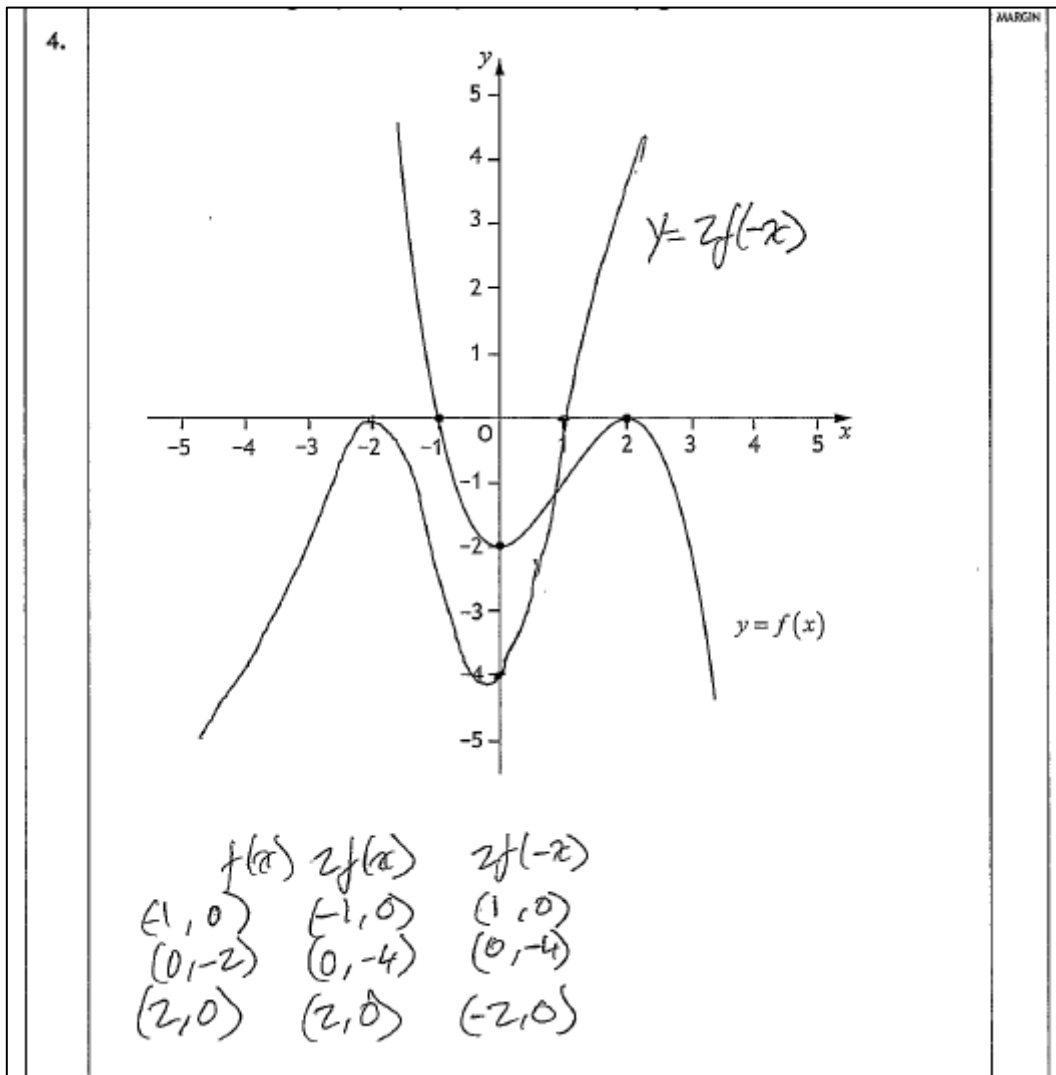
Question 5

Candidate 15

$f(x)(3-2x)^4$

$f'(x) = 4(3-2x)^3 \times 2$

$= 8(3-2x)^3$

$f'(4) = 8(3-2(4))^3$

$= 1000$

Candidate 16

$f(x) = (3-2x)^4$

$f'(x) = 4(3-2x)^3 \times -2$

$= -8(3-2x)^3$

$= -8(3-2(4))^3$

$= -8(5)^3$

$= -8(125)$

$= -1000$

Candidate 17

$f(x) = (3-2x)^4$

$f'(x) = 4(3-2x)^3$

$= -8(3-2x)^3$

$f'(4) = -8(3-2(4))^3$

$= -5000$

Question 6

Candidate 18

6.

$$f(x) = \frac{2}{x} + 3$$

$$y = \frac{2}{x} + 3$$

$$y - 3 = \frac{2}{x}$$

$$x(y-3) = 2$$

$$x = \frac{2}{(y-3)}$$

$$y = \frac{2}{(x-3)}$$

$$f^{-1}(x) = \frac{2}{(x-3)}$$

Candidate 19

6.

$$f(x) = \frac{2}{x} + 3$$

$$y = \frac{2}{x} + 3$$

$$y - 3 = \frac{2}{x}$$

$$\frac{y-3}{2} = x$$

$$f^{-1}(x) = \frac{x-3}{2}$$

Candidate 20

6.

$$f(x) = \frac{2}{x} + 3 \qquad\qquad \frac{2}{x} = \frac{2}{1}x = 2x^{-1}$$

$$y = \frac{2}{x} + 3$$

$$\frac{2}{x} + 3 = y$$

$$\frac{2}{x} = y - 3$$

$$2x^{-1} = y - 3$$

$$x^{-1} = \frac{y - 3}{2}$$

$$x = \frac{y - 3}{2}$$

$$f^{-1}(x) = \frac{x - 3}{2}$$

Candidate 21

6.

$$y = 2x^{-1} + 3 \qquad y = \frac{2}{x} + 3 \qquad xy = 2 + 3$$

$$y - 3 = 2x \qquad y - 3 = \frac{2}{x} \qquad x = \frac{5}{y}$$

$$x = 2 + 3 - y \qquad f^{-1}(x) = \frac{5}{x}$$

$$x = 5 - y$$

Question 7

Candidate 22

7. $\sin x^\circ + 2 = 3\cos 2x^\circ$

$\sin x + 2 = 3 \times 1 - 2\sin^2 x$

$\sin x + 2 = 3 - 2\sin^2 x$

$2\sin^2 x + \sin x - 1 = 0$

$(2\sin x - 1)(\sin x + 1)$

$\sin x = \dfrac{1}{2}$ $\sin x = -1$
                                      reject

$x = 30$

Candidate 23

7.

$$\sin x + 2 = 3\cos 2x$$

$$\sin x + 2 - 3\cos 2x = 0$$

~~$\sin x + 2 - 3(2\cos^2 x - 1) = 0$~~

~~$\sin x + 2 - 6\cos^2 x$~~

$$\sin x + 2 - 3(1 - 2\sin^2 x) = 0$$

$$\sin x + 2 - 3 + 6\sin^2 x = 0$$

$$6\sin^2 x + \sin x - 1 = 0$$

$$(3\sin x + 1)(2\sin x - 1) = 0$$

$$\sin x = -\frac{1}{3} \qquad \sin x = \frac{1}{2}$$

$$ba = 19.5 \qquad ba = 30$$

$$x = 199.5, 340.5 \qquad x = 30, 150$$

$$x = 30, 150, 199.5, 340.5$$

Question 8

Candidate 24

8.

$$\int_{-2}^{1} x^3 - 2x^2 - 4x + 1 \, dx - \int_{-2}^{1} x - 5 \, dx$$

$$\int_{-2}^{1} x^3 - 2x^2 (-4x) + 1 - (x + 5) \, dx$$

$$\int_{-2}^{1} x^3 - 2x^2 - 5x + 6 \, dx$$

$$\left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{5x^2}{2} + 6x \right]_{-2}^{1}$$

$$\left[\frac{1^4}{4} - \frac{2(1)^3}{3} - \frac{5(1)^2}{2} + 6(1) \right] - \left(\frac{2^4}{4} - \frac{2(-2)^3}{3} - \frac{5(2)^2}{2} + 6(-2) \right)$$

$$\left[\frac{1}{4} - \frac{2}{3} - \frac{5}{2} + 6 \right] - \left(\frac{16}{4} + \frac{16}{3} + \frac{20}{2} - 12 \right)$$

$$\left(\frac{3}{12} - \frac{8}{12} - \frac{30}{12} + \frac{72}{12} \right) - \left(\frac{48}{12} + \frac{64}{12} + \frac{240}{12} - \frac{144}{12} \right)$$

$$\left(\frac{37}{12} \right) \quad - \quad \left(\frac{112}{12} \right)$$

$$\frac{-75}{12}$$

$$-6.25 \quad \times -1$$

$$\underline{6.25 \text{ units}^2}$$

$4 \alpha \, z = 8$
$9 \alpha \, z = 14$

Candidate 25

8.

$$(x^3 - 2x^2 - 4x + 1) - (x - 5)$$

$$= \int_{-2}^{1} (x^3 - 2x^2 - 5x + 6)\,dx$$

$$= \left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{5x^2}{2} + 6x\right]_{-2}^{1}$$

$$= \left(\frac{1^4}{4} - \frac{2\times 1^3}{3} - \frac{5\times 1^2}{2} + 6\times 1\right)$$

$$- \left(\frac{-2^4}{4} - \frac{2\times(2)^2}{3} - \frac{5\times(2)^2}{2} + 6(2)\right)$$

$$= \left(\frac{1}{4} - \frac{2}{3} - \frac{5}{2} + 6\right) - \left(4 - \frac{-16}{3} - \frac{10}{2} + (12)\right)$$

$$= \left(\frac{1}{4} - \frac{2}{3} - \frac{5}{2} + 6\right) - \left(4 + \frac{16}{3} - 5 + (12)\right)$$

$$\left(-\frac{5}{12} - \frac{5}{2} + 6\right) - \left(-13 + \frac{16}{3}\right)$$

$$\left(-\frac{35}{12} + 6\right) - \left(-\frac{23}{3}\right)$$

$$\left(\frac{37}{12}\right) - \left(-\frac{23}{3}\right) = \frac{43}{4}$$

Candidate 26

QUESTION NUMBER

DO NOT WRITE IN THIS MARGIN

8.

$$x^3 - 2x^2 - 4x + 1 = x - 5$$

$$x^3 - 2x^2 - 5x + 4 = 0$$

$$\int_{-2}^{1} x^3 - 2x^2 - 5x + 4 \, dx$$

$$\left[\frac{x^4}{4} - \frac{2}{3}x^3 - \frac{5}{2}x + 4x \right]_{-2}^{1}$$

$$\left(\frac{1^4}{4} - \frac{2}{3}(1)^3 - \frac{5}{2}(1) + 4(1) \right) - \left(\frac{(-2)^4}{4} - \frac{2}{3}(-2)^3 - \frac{5}{2}(-2) + 4(-2) \right)$$

$$= \frac{13}{12} - \frac{19}{3}$$

$$= \frac{13}{12} - \frac{76}{12} = \frac{-63}{12}$$

$$= \frac{21}{4} \text{ units}^2 = 5.25 \text{ units}^2$$
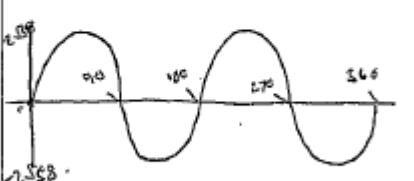
Candidate 27

8.

Curve − line

$$\int_{2}^{1} (x^3 - 2x^2 - 4x + 1) - (x - 5) \cdot dx$$

$$\int_{-2}^{1} (x^3 - 2x^2 - 4x + 1) - x + 5 \cdot dx$$

$$\int_{-2}^{1} x^3 - 2x^2 - 5x + 6 \cdot dx$$

$$\left[\frac{3x^4}{4} - \frac{2x^3}{3} - \frac{5x^2}{2} + 6x \right]_{-2}^{1}$$

$$\left(\frac{3(1)^4}{4} - \frac{2(1)^3}{3} - \frac{5(1)^2}{2} + 6(1) \right) - \left(\frac{3(2)^4}{4} - \frac{2(2)^3}{3} - \frac{5(2)^2}{2} + 6(2) \right)$$

$$= \frac{43}{12} - \left(\frac{-14}{3} \right)$$

$$= \frac{33}{4} \text{ units}^2$$
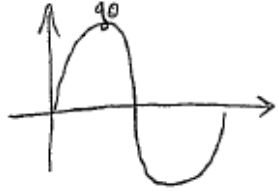
Question 9

Candidate 28

9.(a)

$$7\cos x - 3\sin x = k\sin(x+\alpha)^\circ$$

$$7\cos x - 3\sin x = k\sin x\cos\alpha + k\cos x\sin\alpha$$

$$k\cos\alpha = -3 \qquad\qquad \tan\alpha = \frac{k\sin\alpha}{k\cos\alpha}$$

$$k\sin\alpha = 7 \qquad\qquad \tan\alpha = \frac{7}{-3}$$

$$\qquad\qquad\qquad \alpha = \tan^{-1}\left(\frac{7}{3}\right)$$

$$K = \sqrt{-3^2 + 7^2} \qquad\qquad \alpha = 66.8^\circ$$

$$= \sqrt{40} \qquad\qquad \alpha = 180^\circ - 66.8^\circ$$

$$\qquad\qquad\qquad = 113.2$$

$$7\cos x - 3\sin x = \sqrt{40}\cdot\sin(x+113.2)^\circ$$

Candidate 29

QUESTION NUMBER

DO NOT WRITE IN THIS MARGIN

9.(a)

$7\cos x - 3\sin x = k\sin(x+a)$

$\qquad\qquad = k\sin x \cos a - k\cos x \sin a$

$k\cos a = 7$ $\tan a = \dfrac{\sin a}{\cos a}$

$k\sin a = 3$ $= \dfrac{3}{7}$

$R^2 = 3^2 + 7^2$

$\quad = 9 + 49$ $a = \tan^{-1}\left(\dfrac{3}{7}\right)$

$\quad = 58$ $= 23.2°$

$R = \sqrt{58}$

$7\cos x - 3\sin x = \sqrt{58}\sin(x + 23.2°)$

9.(b)(i)

$14\cos x - 6\sin x = 2(7\cos x - 6\sin x)$

$\qquad\qquad = 2(\sqrt{58}\sin(x + 23.2°))$

$\qquad\qquad = 2\sqrt{58}\sin(x + 23.2°)$

max. value at $\underline{\underline{2\sqrt{58}}}$

9.(b)(ii)

$\leftarrow 23.2$

$2\sqrt{58}\sin(x + 23.2) = 0$

$\sin x + 2.32 = 0, 90°, 180°, 270°, 360$

$x = -2.32, \quad 87.68, \quad 177.68, \quad 267.68, \quad 357.8$

$x < 0, \quad x = 87.68, 177.68, 267.68, 357.8°$

Candidate 30

9.(a)

$7\cos x° - 3\sin x°$ $k\sin(\overset{a}{x} + \overset{b}{a})°$

$K\sin x \cos a + K\cos x \sin a$

$= K \frac{\cos a \sin x + K \sin a \cos x}{}$

$K\sin a = 3$ $\tan a = \dfrac{\sin a}{\cos a}$

$K\cos a = 7$

$K = \sqrt{(7)^2 + (3)^2}$ $\tan a° = \dfrac{-3}{7}$

$= \sqrt{49 + 9}$ $a° = \tan^{-1}\left(\dfrac{-3}{7}\right)$

$= \sqrt{58}$ $= 23 \cdot \dfrac{14}{2}$

$\sqrt{58}\sin(x - 23.2)$

9.(b) (i)

Max value $= \sqrt{58}$

$= 7 \cdot 62$

9.(b) (ii)

accurate

$x = 113 \cdot 2$

$90 + 23 \cdot 2$

$= 113 \cdot 2$
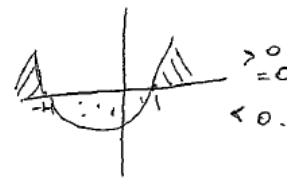
Question 10

Candidate 31

10. $f(x) = 2x^3 + 9x^2 - 24x + 6$

$f'(x) = 6x^2 + 18x - 24 < 0$ for decreasing

     $6(x^2 + 3x - 4) < 0$

     $6(x + 4)(x - 1) < 0$ $(x + 4)(x - 1)$

       $-4 < x < 1$ $x^2 - x + 4x - 4$

Candidate 32

10. $f(x) = 2x^3 + 9x^2 - 24x + 6$ $f'(x) > < 0$.

$f'(x) = 6x^2 + 18x - 24$.

   decreasing when $f'(x) < 0$.

     $6x^2 + 18x - 24 < 0$.

roots $6(x^2 + 3x - 4) = < 0$.

     $6(x + 4)(x - 1) = < 0$.

       $x + 4 = < 0$ $x - 1 \leq 0$.

         $x = < -4$ $x = < 1$.

      when $6x^2 + 18x - 24 < 0$.

        $-4 < x < 1$.

Candidate 33

QUESTION NUMBER

DO NOT WRITE IN THIS MARGIN

10.

$f(x) = 2x^3 + 9x^2 - 24x + 6$

$f'(x) = 6x^2 + 18x - 24$

$6x^2 + 18x - 24 \lessgtr 0$

$6(x^2 + 3x - 4) \lessgtr 0$

$6(x + 4)(x - 1) \lessgtr 0$

$x \lessgtr -4 \; , \; x \lessgtr 1$

When $x = -4$

$y = 2x^3 + 9x^2 - 24x + 6$

$y = 2(-4)^3 + 9(-4)^2 - 24(-4) + 6$

$y = 118$

When $x = 1$

$y = 2(1)^3 + 9(1)^2 - 24(1) + 6$

$= 2 + 9 - 24 + 6$

$= -7$

x	$\xrightarrow{-5}$	-4	$\xrightarrow{0}$	1	$\xrightarrow{2}$
$\dfrac{dy}{dx}$	$+$	0	$-$	0	$+$
Slope	/	—	\	—	/

$(x + 4)(x - 1)$

$- \quad -$

$+ \quad -$

$+ \quad +$

maximum t·p when at $(-4 , 118)$

minimum t·p when at $(1 , -7)$

Candidate 34

10. $f'(x) = \cancel{12x} 6x^2 + 18x - 24$

$6x^2 + 18x - 24 = 0$

$6(x^2 + 3x - 4) = 0$

$6(x^2 + 4x - x - 4) = 0$

$6(x(x+4) - 1(x+4)) = 0$

$6(x-1)(x+4) = 0$

$x = 1 \quad x = -4$

	→	-4	→	1	→
$f'(x) = 6x^2 + 18x - 24$	+	0	−	0	+
Slope	/	―	\	_	/

x is strictly decreasing when $-4 < x < 1$

Candidate 35

QUESTION NUMBER

10.

$f'(x)$

$\frac{dy}{dx} < 0$ for decreasing

$f'(x) = 6x^2 + 18x - 24$

$6(x^2 + 3x - 4) < 0 \qquad \frac{x \mid 4}{x \mid -1}$

$(x+4)(x-1)$

$x < -4 \qquad x < 1$

$-4 < x > 1$

Question 11

Candidate 36

| QUESTION NUMBER | | DO NOT WRITE IN THIS MARGIN |

11.(a)

$(x-4)^2 + (y+2)^2 = 37$

$\qquad C(4,-2)$

$x^2 + y^2 + 2x - 6y - 7 = 0$

$2g = 2 \qquad 2f = -6$

$g = 1 \qquad f = -3$

$\underset{(4,-2)}{\overset{x_1 \; y_1}{\ }} \qquad \underset{(-1,3)}{\overset{x_2 \; y_2}{\ }} \qquad (-1,3)$

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$= \sqrt{(4-61)^2 + (-2-3)^2}$

$= \sqrt{(5)^2 + (-5)^2}$

$= \sqrt{25 + 25} \qquad \qquad = \sqrt{25}\sqrt{5}$

$= \sqrt{50} \qquad \qquad \qquad = \sqrt{} = 2\sqrt{5}$

11.(b)

$g = 1 \qquad f = -3$

$r = \sqrt{g^2 + f^2 - c} \qquad \qquad r = 6 = -843$

$= \sqrt{1^2 + (-3)^2 + 7} \qquad \qquad r = \sqrt{37}$

$= \sqrt{17}$

$\sqrt{17} + \sqrt{37} = \sqrt{54} \qquad D = \sqrt{50}$

$\qquad \qquad \qquad \qquad = \sqrt{25}\sqrt{5}$

$r_1 + r_2 \neq d \qquad \qquad = 2\sqrt{5}$

$\sqrt{17} - \sqrt{37}$

$\neq 0 \qquad$ Th∴ They do not touch inside the circle

Candidate 37

11.(a) $C_1 (4, -2)$

$C_2 (-1, 3)$

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$d = \sqrt{(4 - (-1))^2 + ((-2) - 3)^2}$

$d = \sqrt{5^2 + (-5)^2}$

$d = \sqrt{25 + 25}$

$d = \sqrt{50}$

11.(b) C_1
radius $= \sqrt{37}$

C_2

$r = \sqrt{1^2 + -3^2 + 7}$

$= \sqrt{1 + 9 + 7}$

$= \sqrt{10 + 7}$

$= \sqrt{17}$

$d < r_1 + r_2$

$\sqrt{50} < \sqrt{17} + \sqrt{37}$

this shows that both circles intercept at two points

Candidate 38

11.(a)

$C_1 \; (x-4)^2 + (y+2)^2 = 37$

$C_2 \; x^2 + y^2 + 2x - 6x - 7 = 0$

$C_1 \text{ centre} = \begin{pmatrix} x_1 & y_1 \\ 4 & -2 \end{pmatrix}$

$C_2 \text{ centre} = \begin{pmatrix} x_2 & y_2 \\ -1 & 3 \end{pmatrix}$

$2gx = 2x$
$g = 1$

$2fy = -6x$
$y = -3$

$(-g, -f) = \text{centre}$
$= (-1, 3)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-1-4)^2 + (3--2)^2}$

$= 5\sqrt{2}$

$= 7 \cdot 07$

11.(b)

intersect at 2 distinct points if $r_1 + r_2 < $ distance between 2 centres

$r_{C_1} = \sqrt{37}$

$r_{C_2} = \sqrt{g^2 + f^2 - c}$

$= \sqrt{(1)^2 + (-3)^2 - (-7)}$

$= \sqrt{1 + 9 - 7}$

$= \sqrt{3}$

$\sqrt{37} + \sqrt{3} = 6 \cdot 22$

$d = 7 \cdot 07$

∴ intersect at more than one point

Question 12

Candidate 39

12.

$$\frac{dy}{dx} = 8x^3 + 3 \qquad (-1, 3)$$

$$y = \frac{8x^4}{4} + 3x$$

$$y = 2x^4 + 3x$$

sub $(-1, 3)$ in

$$3 = 2(-1)^4 + 3(-1)$$

$$3 = -1 +\!\!\!\!\!^3$$

$$c = 4$$

$$y = 2x^4 + 3x + c$$

Candidate 40

12. $\dfrac{dy}{dx} = 8x^3 + 3$ $(-1, 3)$

$\int (8x^3 + 3)\,dx$

$= \dfrac{8x^4}{4} + 3x + c$

$= 2x^4 + 3x + c$

$3 = 2(-1)^4 + 3(-1) + c$

$3 = 2 + 3 + c$

$4 = c$

$2x^4 + 3x + 4$

Candidate 41

12. $\dfrac{dy}{dx} = 8x^3 + 3$

$\int 8x^3 + 3 \; dx$

$= \dfrac{8x^4}{4} + 3x$

$= 2x^4 + 3x + C$

$3 = 2(-1)^4 + 3(-1) + c$

Candidate 42

QUESTION NUMBER

DO NOT WRITE IN THIS MARGIN

12.

$$\frac{dy}{Dx} = 8x^3 + 3 \qquad (-1, 3)$$
$$\qquad\qquad\qquad\qquad x \quad y$$

$$y = \int 8x^3 + 3$$

$$y = \frac{8x^4}{4} + 3x + C$$

$$3 = \frac{8(-1)^4}{4} + 3(-1) + C$$

$$3 = 2 - 3 + C$$

$$3 = -1 + C$$

$$4 = C$$

$$y = \frac{8x^4}{4} + 3x + 4$$

Question 13

Candidate 43

13.(a)

$$C_1 = 11e^{-0.0053(30)}$$

$$C_1 = 11e^{-1.59}$$

$$C_1 = 53.94$$

$$C_1 = 53.9 \text{ mg/l}$$

13.(b)

$$C_1 = 11e^{-0.0053t}$$

$$0.66 = 11e^{-0.0053t}$$

$$\log_e 0.$$

$$\log_e 0.66 = \log_e$$

$$\frac{0.66}{11} = e^{-0.0053t}$$

$$0.06 = e^{-0.0053t}$$

$$\log_e 0.06 = \log_e e^{-0.0053t}$$

$$= -0.0053t$$

$$\frac{-1.12218}{-0.0053} = t$$

$$230.537... = t$$

$$230.5 \text{ minutes} = t$$

$$230.5 \text{ minutes} = t$$

Candidate 44

QUESTION NUMBER		DO NOT WRITE IN THIS MARGIN
13.(a)	$C_t = 11e^{-0.0053t}$ $C_t = 11e^{-0.0053 \times 30}$ $C_t = 9.38 \, mg/l$	
13.(b)	$0.66 = 11e^{-0.0053t}$ $\dfrac{0.66}{11} = e^{-0.0053t}$ $\ln \dfrac{0.66}{11} = \ln e^{-0.0053t}$ $\ln \dfrac{0.66}{11} = -0.0053t \ln e$ $-2.8 = -0.0053t$ $t = 528.3 \, minutes$	

Candidate 45

QUESTION NUMBER		DO NOT WRITE IN THIS MARGIN
13.(a)	$\boxed{Cg} = 11e^{-0.0053t}$	
	$Ct = 11e^{-0.0053 \times 30}$	
	$Ct = \underline{\underline{9.4}}$	
13.(b)	$Ct = 11e^{-0.0053\boxed{t}}$ $\qquad\qquad$ $9.4 - 0.66$	
	$8.74 = 11e^{-0.0053t}$ $\qquad\qquad = 8.74$	
	$11e^{-0.0053t} = 8.74$	
	$e^{-0.0053t} = \dfrac{8.74}{11}$	
	$= 0.7945$	
	$-0.0053t = \ln(0.7945)$	
	$t = \dfrac{\ln 0.7945}{-0.0053}$	
	$t = 43.4 \text{ minutes}$	

Candidate 46

13.(b)

$$C_t = 11e^{-0.053t}$$

$$0.66 = 11e^{-0.053t}$$

$$\frac{0.66}{11} = e^{-0.053t}$$

$$\ln 0.06 = \ln e^{-0.053t}$$

$$-2.81 = -0.053t$$

$$-0.053t = -2.81$$

$$t = \frac{-2.81}{-0.053}$$

$$= 53 \text{ minutes}$$

Question 14

Candidate 47

14.(a) (i)	$A = l \times b$ $3x \times 2x = 6x$ $2(h \times 2x)$ $2(3x \times h)$ $A = 6x + 2(3hx) + 2(2hx)$ $A = 6x + 6hx + 4hx$ $A = 6x + 10hx$
14.(a) (ii)	$A = 6x + 10hx = 7200$ $V = h \times l \times b$ $h = \dfrac{7200}{6x + 10x}$ $10hx = 7200 - 6x$ $h = \dfrac{7200 - 6x}{10x}$ $h = \dfrac{7200 - 3x}{5x}$

QUESTION NUMBER		DO NOT WRITE IN THIS MARGIN

nx^{n-1}

14.(b)

$$V = 4320x - \frac{18}{5}x^3$$

$$V'(x) = 4320 - \frac{54}{5}x^2$$

At Stationary Point $V'(x) = 0$

$$4320 - \frac{54}{5}x^2 = 0$$

$$\frac{54}{5}x^2 = 4320$$

$$x^2 = 400$$

$$x = \sqrt{400}$$

$$\underline{\underline{x = 20}}$$

Candidate 48

QUESTION NUMBER

DO NOT WRITE IN THIS MARGIN

14.(a) (i)

$A1 = 3x \times 2x$
$= 6x^2$

$A4 = 2x \times h$
$= 2xh$

$A2 = 3x \times h$
$= 3xh$

$A5 = 3x \times h$
$= 3xh$

$A3 = h \times 2x$
$= 2xh$

$A = l \times b$

$A total = 6x^2 + 3xh + 2xh + 2xh + 3xh$

$= 6x^2 + 6xh + 4xh$

14.(a) (ii)

$V = 4320x - \frac{18}{5}x^3$

$V'(x) = 4320 - \frac{54}{5}x^2$

$0 = 4320 - \frac{54}{5}x^2$

$\frac{54}{5}x^2 = 4320$

$54x^2 = 21600$

$x^2 = \frac{21600}{54}$

$x^2 = 400$

$x = \sqrt{400}$

$x = 20$

maximises at $x = 20$

Stationary points at

$V'(x) = 0$

$V'(x)$	$< 0 <$		
x	19	20	21
slope	+	—	-
	/	—	\

$V'(19) = 4320 - \frac{54}{5}(19)^2$

$= 427 \cdot 2$

$V'(21) = 4320 - \frac{54}{5}(21)^2$

$= -442 \cdot 8$

14.(b)

$$V = 4320x - \frac{18}{5}x^3$$

$$V'(x) = 4320 - \frac{54}{5}x^2$$

Stationary points at $V'(x) = 0$

$$0 = 4320 - \frac{54}{5}x^2$$

$$\frac{54}{5}x^2 = \frac{21600}{4320}$$

$$54x^2 = 21600$$

$$x^2 = 400$$

$$x = \sqrt{400}$$

$$x = 20$$

$V'(x)$	$\overset{19}{<}$	0	$\overset{21}{<}$
x	$+$	20	$-$
slope	$\diagup$	$-$	$\diagdown$

$$V'(19) = 4320 - \frac{54}{5}(19)^2$$
$$= 421.2$$

$$V'(21) = 4320 - \frac{54}{5}(21)^2$$
$$= -442.8$$

max + pat $x = 20$

14(iii)) $A = 7,200 cm^3$

$$7200 = 6x^2 + 6xh + 4xh$$

$$V = l \times b \times h$$

Candidate 49

14.(b)

$$V = 4320x - \frac{18}{5}x^3$$

$$V' = 4320 - 3\left(\frac{18}{5}\right)x^4$$

$$= 4320 - \frac{54}{5}x^4$$

For SV, $V' = 0$

$$4320 - \frac{54}{5}x^4 = 0$$

$$\frac{54}{5}x^4 = 4320$$

$$54x^4 = 21600$$

$$x^4 = \frac{21600}{54}$$

$$= 400$$

$$x = \sqrt[4]{400}$$

$$= 4.4721\ldots$$

$$= 4.5$$

$$= 4\tfrac{1}{2}$$

x	4	$4\tfrac{1}{2}$	5
V'	+	0	−
shape	/	—	\

∴ a maximum

at $x = 4.5$

Question 15

Candidate 50

15.

$$3y = -x + 17$$

$$y = \frac{-x}{3} + 17$$

$$m = \frac{-1}{3}$$

$$m_\perp = 3$$

$$y - 5 = 3(x - 2)$$

$$y - 5 = 3x - 6$$

$$y = 3x - 1$$

$$3x - 1 = 0$$

$$3\left(x - \frac{1}{3}\right) = 0$$

$$x = \frac{1}{3} \text{ ad } y = 0$$

centre of circle $= \left(\frac{1}{3}, 0\right)$

Candidate 51

15.

$x + 3y = 17$

$3y = 17 - x$

$3(1) = 17 - x$

$3 = 17 - x$

$3 - 17 = -x$

$x = 20$

$P(1, 20)$

At $y = 1$

$x + 3y = 17$

$x = 17 - 3y$

$= 17 - 3$

$= 14$

$P(1, 14)$

For $\perp$ lines
$m_1 m_2 = -1$

$\therefore m_\perp = \frac{1}{a}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{14 - 5}{1 - 2}$

$= -\frac{a}{1}$

$= -a$

$m = \frac{1}{a}$

$\frac{1}{a} = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{5 - y_1}{2 - 0}$

$\frac{2}{a} = 5 - y_1$

$\frac{2}{45} = -y_1$

$y_1 = -\frac{2}{45}$

$P\left(0, -\frac{2}{45}\right)$

Candidate 52

15.

$$x + 3y = 17$$

$$x + 3(0) = 17$$

$$x = 17 \quad (17, 0)$$

$$m = \frac{5 - 0}{2 - 17}$$

$$= -\frac{5}{15}$$

$$= -\frac{1}{3}$$

$$m \perp = 3$$

$$3 = \frac{5 - x}{2 - y}$$

$$3 = \frac{5 - x}{2 - 0} \qquad 3 = \frac{5 - 0}{2 - y}$$

$$6 = 5 - x \qquad 6 - 3y = 5$$

$$6 = 5 + 3y$$

$$1 = 3y$$

$$y = \frac{1}{3}$$

$$\left(0, \frac{1}{3}\right)$$