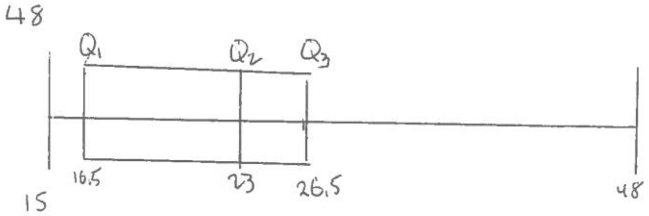
Candidate 1 evidence

16.5 26.5

1a) 15 15 16 | 17 20 22 |23| 23 25 26 | 27 30



$IQR = 26.5 - 16.5$
$= 10$

b) Upper limit $= Q_3 + 1.5 \times IQR$

$= 26.5 + 1.5 \times 10$

$= 41.5$

∴ 48cm is an outlier of the data

2a) There may be an underlying pattern in the selected patients

The patients may not be randomly distributed and so uneven amounts of ~~exa~~ similar patients might be obtained.

b) A more representative sample could be obtained by using stratified random sampling in which you divide the patients into strata according to certain traits (male, female etc.) and use random sampling to select the patients to be used from each strata.

c)	Another possible cause of variation could be how long the patient had to wait to. recieve surgery.

Candidate 2 evidence

3. a) X = weight of jar (g)

$X \sim N(\mu, 2{\cdot}379^2)$

$H_0: \mu = 150$

$H_1: \mu < 150$

Assume H_0 to be true

$\alpha = 1\%$, one-tail test

$X \sim N(150, 2{\cdot}379^2)$

$\bar{X} \sim N\left(150, \frac{2{\cdot}379^2}{12}\right)$ where $\bar{X}$ = mean weight of 12 jars

$$p\text{-value} = P(\bar{X} < 147{\cdot}8)$$

$$= P\left(Z < \frac{147{\cdot}8 - 150}{\sqrt{\frac{2{\cdot}379^2}{12}}}\right)$$

$$= P(Z < -3{\cdot}20346)$$

$$= 0{\cdot}000679$$

$$<< 0{\cdot}01$$

we are within critical region ∴ have evidence to reject H_0 and conclude that the mean weight is less than 150g

b). n is too small ($n > 20$ for z-test).

Candidate 3 evidence

4.a) i) the data is skewed

ii) $E(x) = \frac{\Sigma x}{n} = \frac{142}{20} = 7.1$

$$V(x) = \frac{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}{n-1} = \frac{1120.16 - \frac{142^2}{20}}{20-1} = 5.892631579$$

$$SD(x) = \sqrt{V(x)} = \sqrt{5.892631579} = 2.427474321$$
$$= 2.427$$

b)

$$\cancel{P(x > 8.1) = 1 - P(x \leq 8.1)}$$
$$\cancel{= 1 - P\left(Z \leq \frac{8.1 - 7.1}{(2.427)}\right)}$$
$$\cancel{= 1 - P\left(Z \leq \frac{1}{2.427}\right)}$$
$$\cancel{= 1 - \Phi(0.41)}$$
$$\cancel{=}$$

$$P(x > 8.1) = 1 - P(x \leq 8.1)$$
$$= 1 - P\left(Z \leq \frac{8.1 - 7.1}{2.427}\right)$$
$$= 1 - P(Z \leq 0.41)$$
$$= 1 - \Phi(0.41)$$
$$= 1 - 0.6591$$
$$= 0.3409$$

Candidate 4 evidence

5a) i) $\bar{x} = \underline{4.75}\text{kg}$ $s = 0.46$

confidence interval $= \bar{x} \pm t_{n-1, 1-\frac{\alpha}{2}} \left(\frac{s}{\sqrt{n}}\right)$

$= (4.46 , 5.04)$

ii) This confidence interval does not support this suggestion as 4.75 kg lies within the confidence interval, meaning the mean yield could potentially lie below 4.75kg

b) The weather could vary in that some days it may be more over-cast, not allowing for more light for the tomato plants and some plants might be in better lighting conditions

Candidate 5 evidence

6. a) Before month 14 the majority of points fall above 0·025 of patients, before then after 15 months of decreasing to primarily below 0·025, indicating that something has occurred to cause a decrease.

b) $X = n^{o}$ times point is above / below the line

$X \sim Bin(14, \frac{12}{14})$

$P(X = 12) = 0{\cdot}292068$ from binompdf $(14, \frac{12}{14}, 12)$

$\approx 0{\cdot}2921$ (4d.p)

c) 14 and 16

d) $UCL = 0{\cdot}025 + 3 \times \sqrt{\overline{n}}$ total ops $= 30 \times 975$

$= 29\,250$

e) as n is large and p is small you are guaranteed $np > 5$, $nq > 5$ so it will be a good approximation

$x = $ ~~no~~ prop patients w/ difficult recovery

~~$x \sim Bin(29250, p)$~~

~~[illegible]~~ $x \sim Bin(30, 0.025)$

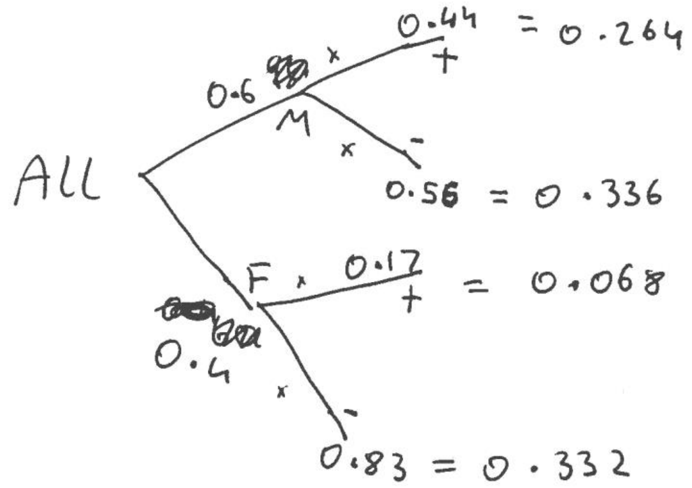
$P(x > 0.03) = 1 - P(x \leq 0.03)$ from binomcdf $(30, 0.025, 0, 0.03)$

$= 0.532116$

$\therefore \approx 53\%$

Candidate 6 evidence

7.



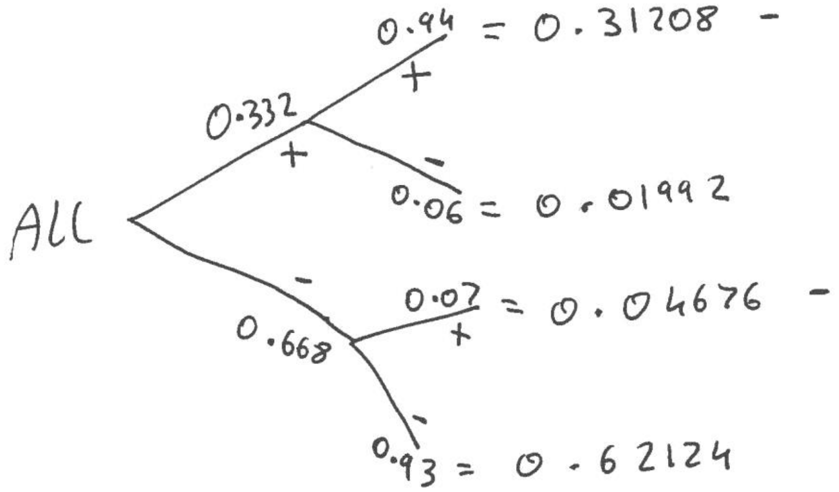
a) $p(\text{Virus}) = \dfrac{0.068 + 0.264}{0.068 + 0.264 + 0.332 + 0.336}$

$= \underline{\underline{0.332}}$

b) $p(3 \text{ disease}) = (0.332)^3$

$= \underline{\underline{0.0366}}$

7 c i)



$P(\text{+ive reaction}) = 0 \cdot 31208 + 0 \cdot 04676$

$= \underline{\underline{0 \cdot 35884}}$

c ii) $P(\text{Virus}/\text{+'ve reaction}) = \dfrac{0 \cdot 31208}{0 \cdot 31208 + 0 \cdot 04676}$

$= \underline{\underline{0 \cdot 8697}}$

Candidate 7 evidence

8 a) Validity: two sample, non-paired, proportions.

Assuming recovery rates for the two types of drugs are normally distributed.

X_1 = new
$p_1 = 0.75$
$n_1 = 100$

X_2 = control
$p_2 = 0.65$
$n_2 = 100$

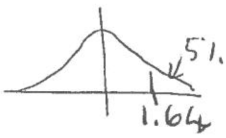
$H_0: p_1 = p_2$
$H_1: p_1 > p_2$

Assume H_0 to be true. $\alpha = 5\%$ one-tail test

$$p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{75 + 65}{200} = \frac{7}{10}$$

$$\therefore \frac{p_1 - p_2}{\sqrt{pq\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0,1)$$

$$Z = \frac{0.75 - 0.65}{\sqrt{\frac{7}{10} \times \frac{3}{10}\left(\frac{1}{100} + \frac{1}{100}\right)}} = 1.54303$$



As $1.543 < 1.64485$

we are not in critical region and conclude the new drug did not have a higher recovery rate.

b) The new drug could actually produce a lower recovery rate than the control so want to test whether its effect is significant either way.

Candidate 8 evidence

9.a) the sporting injuries are randomly distributed and are independent

b) $P(X > 2) = 1 - P(X \leq 2)$

$= 1 - \left(\frac{e^{-4} \times 4^{2}}{2!} + \frac{e^{-4} \times 4^{1}}{1!} + \frac{e^{-4} \times 4^{0}}{0!} \right)$

$= 1 - 0.2381$

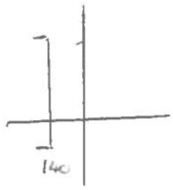
$= 0.7619$

c) ~~mean~~ sports injuries in 38 weeks $= 38 \times 4$
$= 152$

$P(4) \simeq N(4,4) \Rightarrow P(152) \simeq N(152,152)$

$P(X < 140) = P(X \leq 140)$

$= P\left(Z \leq \frac{140 - 152}{\sqrt{152}}\right)$



$= P(Z \leq -0.9733285268)$

$= \Phi(-0.97)$

$= 1 - \Phi(0.97)$

$= 1 - 0.8340$

$= 0.1660$

d) sporting injuries are random and ~~cons~~ ~~thus~~ therefore the mean sporting injuries per week is not a reliable comparison, a ~~poiss~~ poisson distribution is therefore no appropriate

Candidate 9 evidence

10(a) x 1 2

$E(X) = \sum x P(X=x)$

$\frac{7}{4} = P(X=1) + 2P(X=2)$

$V(X) = \sum E(X^2) - \frac{(E(X))^2}{\cancel{}}$

$\cancel{= P(X=1) + 4P(X=2)}$

let $P(X=1) = A$ $P(X=2) = B$

$\frac{7}{4} = A + 2B$ $\frac{3}{16} = A + 4B - \frac{(A+2B)^2}{\cancel{}}$

$A = \frac{7}{4} - 2B$

$\cancel{= A - \frac{A^2}{2} + 4B + 4B^2 + 2AB}$

$= A + 4B - A^2 - 4AB - B^2$

$= \frac{7}{4} - 2B + 4B - \left(\frac{7}{4} - 2B\right)^2$

$- 4B\left(\frac{7}{4} - 2B\right) - B^2$

$-\frac{25}{16} = 2B - \frac{49}{16} - 4B^2 + 7B$

$-7B + 8B^2 - B^2$

$24 = 2B - 4B^2 + 8B^2 - B^2$

$= 2B + 3B^2$

$\frac{-b \pm \sqrt{b^2 - 4a}}{2a}$

at back.

10(a)

$E(X) = \frac{7}{4} \quad V(X) = \frac{3}{16}$

$E(X) = \sum x P(X=x)$

$\frac{7}{4} = 1 \times P(X=1) + 2 \times P(X=2)$

~~$P(X=1) = \frac{7}{12}$ $P(X=2) =$~~

let $P(X=1) = A \quad B = P(X=2)$

$\frac{7}{4} = A + 2B \quad \therefore \quad A = \frac{7}{4} - 2B$

$V(X) = E(X^2) - (E(X))^2$

$\frac{3}{16} = A + 4B - (A+2B)^2$

$= \frac{7}{4} - 2B + 4B - \left(\frac{7}{4} - 2B + 2B\right)^2$

$= \frac{7}{4} + 2B - \left(\frac{7}{4}\right)^2$

$= \frac{7}{4} + 2B - \frac{49}{16}$

$2B = \frac{3}{2}$

$B = \frac{3}{4}$

$A = \frac{7}{4} - 2B$

$= \frac{7}{4} - \frac{6}{4}$

$= \frac{1}{4}$

x	1	2
$P(X=x)$	$\frac{1}{4}$	$\frac{3}{4}$

10(b)

$E(Y) = \sum x P(X=x)$

$= 1 \times \frac{2}{5} + 2 \times \frac{3}{5}$

$= \frac{2}{5} + \frac{6}{5}$

$= \frac{8}{5}$

$= 1.6$

$V(Y) = E(X^2) - (E(X))^2$

$= 1 \times \frac{2}{5} + 4 \times \frac{3}{5} - 1.6^2$

$= \frac{14}{5} - 1.6^2$

$= 0.24$

10(c)

$E(3X - Y)$

$= 3E(X) - E(Y)$

$= 3 \times \frac{7}{4} - 1.6$

$= 3.65$

$V(3X - Y)$

$= 3^2 V(X) + V(Y)$

$= 9V(X) + V(Y)$

$= 9 \times \frac{3}{16} + 0.24$

$= 1.9275$

$= 1.93$

Candidate 10

11. (a) $H_0 : \mu_d = 0$ (μ_d = mean difference)

$H_1 : \mu_d > 0$

$\bar{d} = 1.175$

$s_d = 4.272$

$n = 12$

t-test statistic $= \dfrac{\bar{d} - 0}{\frac{4.272}{\sqrt{12}}}$

$= 0.953$

dof $= n - 1$

$= 11$

critical value at 5% = 1.796

As $t < 1.796$ there is insufficient evidence to reject H_0 at the 5% level. There is insufficient evidence to suggest that the mean height for offspring bred via cross pollination is greater than the mean height for offspring bred via self-pollination.

(b) A wilcoxon test for paired data could be employed along with the assumption that each pair of plants growth is independent of each other.

$H_0 : \eta_d = 0$

$H_1 : \eta_d > 0$ (η_d = median difference)

Diff :	6.1	−8.4	1.0	0.0	2.0	2.9	3.5	5.1	1.8
	10	11	1	/	3	4	7	8	2

Diff :	3.1	3.0	−6.0
	6	5	9

$W_- = 20$

$W_+ = 46$

W, test statistic = 20

$n = 11$

critical value at 5% = 13

As $W > 13$ there is insufficient evidence to reject H_0 at the 5% level. There is insufficient evidence to suggest the mean offspring height bred via cross pollination is greater than the mean height of offsprings bred via self pollination.

Both tests result in the same conclusion of insufficient evidence to reject H_0 at the 5% level. There may be an indication that the underlying population is normally distributed.

(c) A benefit of having a paired study design will be that ~~each~~ both plants in each pair will have been grown in the same conditions. Limiting any outside factors that could occur.

Candidate 11

11a) $H_0: \bar{d} = 0$ $H_1: \bar{d} > 0$ $\therefore$ 1-tail test
$\alpha = 0.05$

$\bar{d} = 1.175$
$n = 12$
$s = 4.272$

Observed value: $t = \dfrac{\bar{d} - 0}{\frac{s}{\sqrt{n}}}$

$= \dfrac{1.175}{\frac{4.272}{\sqrt{12}}}$

$= 0.95$

critical value: $+ t_{n-1,\ 0.95}$

$= + t_{11,\ 0.95}$

$= 1.796$

critical region: $t > 1.796$

$0.95 \not> 1.796 \quad \therefore$ do not reject H_0

we do not have enough evidence at the 5% level to suggest $\bar{d} > 0$ (mean height of offspring bred via cross pollination is greater than mean height of offspring bred via ~~cro~~ self pollination)

$H_0: \eta_{cross} = \eta_{self}$

11b) Wilcoxon signed rank test

$H_1: \eta_{cross} > \eta_{self}$ (1-tail) $\therefore \alpha = 0.05$

~~assum~~

~~difference 0 1 1.8 2 2.4 3 3.1 3.5 5.1 6~~
~~rank~~

difference	~~0~~	1	1·8	2	2·4	3	3·1	3·5	5·1	−6	6·1	−8·4
rank		1	2	3	4	5	6	7	8	9	10	11

$W_- = 20$

$n = 11$

Observed value: $W_- = 20$

critical value: $W = 13$

$20 \not< 13 \quad \therefore$ do not reject H_0

We do not have enough evidence at the 5% level to suggest $\eta_{cross} > \eta_{self}$ as in part a)

assumption: distributions of cross pollination and self pollination have similar shape

11c) it's easier to identify differences between two sets of data ~~rather than using~~ by being given the sets rather than using historical mean μ and sample mean $\bar{x}$

Candidate 12

12\. a) The more attractive the rider the better the perfomance

b) $r = \dfrac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

0 – .2	weak
.2 – .4	mild
→ .4 – .6	~~moderate~~ moderate
.6 – .8	moderately strong
.8 – 1	strong

$r = \dfrac{15.6348}{\sqrt{58.3111 \times 26.1816}}$

$r = 0.400146130 3$

~~[illegible]~~

this measures the strength of a correlation. The value of 0.4001461303 suggests a moderate correlation between attractiveness and performance

c) H_0: there is no correlation between attractiveness and performance

H_1: there is a correlation between attractiveness and performance

Two tailed $\alpha = 0.05$ $\pm t_{68,\,0.975} = \pm t_\infty = 1.96$

$t = \dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}}$

$t = \dfrac{0.4001461303\sqrt{68}}{\sqrt{1-0.4001461303^2}}$

$t = 3.6$

Since t lies outwith the confidence interval $\pm t_\infty$ we can reject H_0. There is evidence to suggest that there is a correlation between attractiveness and performance.

d) a small correlation ~~[illegible]~~ allows a $\sqrt{1-r^2}$ value to be ~~[illegible]~~ low enough so that the overall test statistic is high when $r \approx 0.5$

The final conclusion to this study would be that since ~~[illegible]~~ the test statistic lies outwith the 5% confidence interval we can reject H_0. There is significant evidence to suggest that there is a correlation between ~~height~~ performance and attractiveness, leading to evidence that the correlation coefficient is ~~[illegible]~~ significantly different from zero.

e) the linear model is $y_i = \alpha + \beta x_i + \varepsilon_i$ where ε_i are independent, $E(\varepsilon_i) = 0$ and $V(\varepsilon_i) = \sigma^2$