Question 7

Response 1

7) $a = 4 - t^{1/2}$

max speed when $a = 0$

$0 = 4 - \sqrt{t}$

$\sqrt{t} = 4$

$t = 4^2$

$t = 16$ s

$v = \int a \, dt$

$v = 4t - \frac{2}{3}t^{3/2}$

Let $t = 16$

$v = 4 \times 16 - \frac{2}{3} \times 16^{3/2}$

$v = \frac{64}{3}$ m/s

$v = 21.3$ m/s

Inc E_k initially: $v = 0$

$E_k = \frac{1}{2} \times g \times 0^2$

$= 0$ J

E_k at $t = 16$

$E_k = \frac{1}{2} \times g \times \left(\frac{64}{3}\right)^2$

$E_k = 2048$ J

Increase of 2048 J

Question 8 (b)

Response 2

ENTER NUMBER OF QUESTION 8.

b) $\frac{3x^3+8x^2-11}{(x+1)(x+3)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x+3)} + \frac{C}{(x-2)}$

$3x^3+8x^2-11 = A(x+3)(x-2)+B(x+1)(x-2)+C(x+1)(x+3)$

$x=-1$: $-3+8-11=-6A$ $\quad -6=-6A$ $\quad \boxed{A=1}$

$x=2$: $24+32-11=15C$ $\quad 15C=45$ $\quad \boxed{C=3}$

$x=-3$: $-81+72-11=10B$ $\quad -20=10B$ $\quad \boxed{B=-2}$

$\frac{3x^3+8x^2-11}{(x+1)(x+3)(x-2)} = \frac{1}{x+1} - \frac{2}{x+3} + \frac{3}{x-2}$

Response 3

ENTER NUMBER OF QUESTION b.

$3 + \frac{2x^2+15x+7}{(x+1)(x+3)(x-2)} = \frac{A}{(x+1)} + \frac{B}{(x+3)} + \frac{C}{(x-2)}$

$3(x+1)(x+3)(x-2)+2x^2+15x+7 = A(x+3)(x-2)+B(x+1)(x-2)+C(x+1)(x+3)$

$3(x^3+2x^2-5x-6)+2x^2+15x+7 = A(x+3)(x-2)+B(x+1)(x-2)+C(x+1)(x+3)$

$x=-1$ $\quad -6=A(2)(-3)$ $\quad -6=-6A$ $\quad A=1$

$x=-3$ $\quad -20=B(-2)(-5)$ $\quad -20=10B$ $\quad B=-2$

$x=2$ $\quad 45=C(3)(5)$ $\quad 45=15C$ $\quad C=3$

$\frac{3x^3+8x^2-11}{(x+1)(x+3)(x-2)} = 3 + \frac{1}{x+1} - \frac{2}{x+3} + \frac{3}{x-2}$

Question 15 (b)

Response 4

b

$\frac{dv}{dx} + 24v + 80x = 0$ $y = Ae^{mx}$ is a solution

$\frac{dv}{dx} = -24v - 80x$ $Ae^{mx}(m^2 + 24m + 80) = 0$

$(m + 20)(m + 4) = 0$

$m = -20, -4$

$y = Ae^{-20x} + Be^{-4x}$

$0 = Ae^{-20 \times 0.2} + Be^{-4 \times 0.2}$

$0.2 = -2A + B$

$0.2 = -20A - 4B$

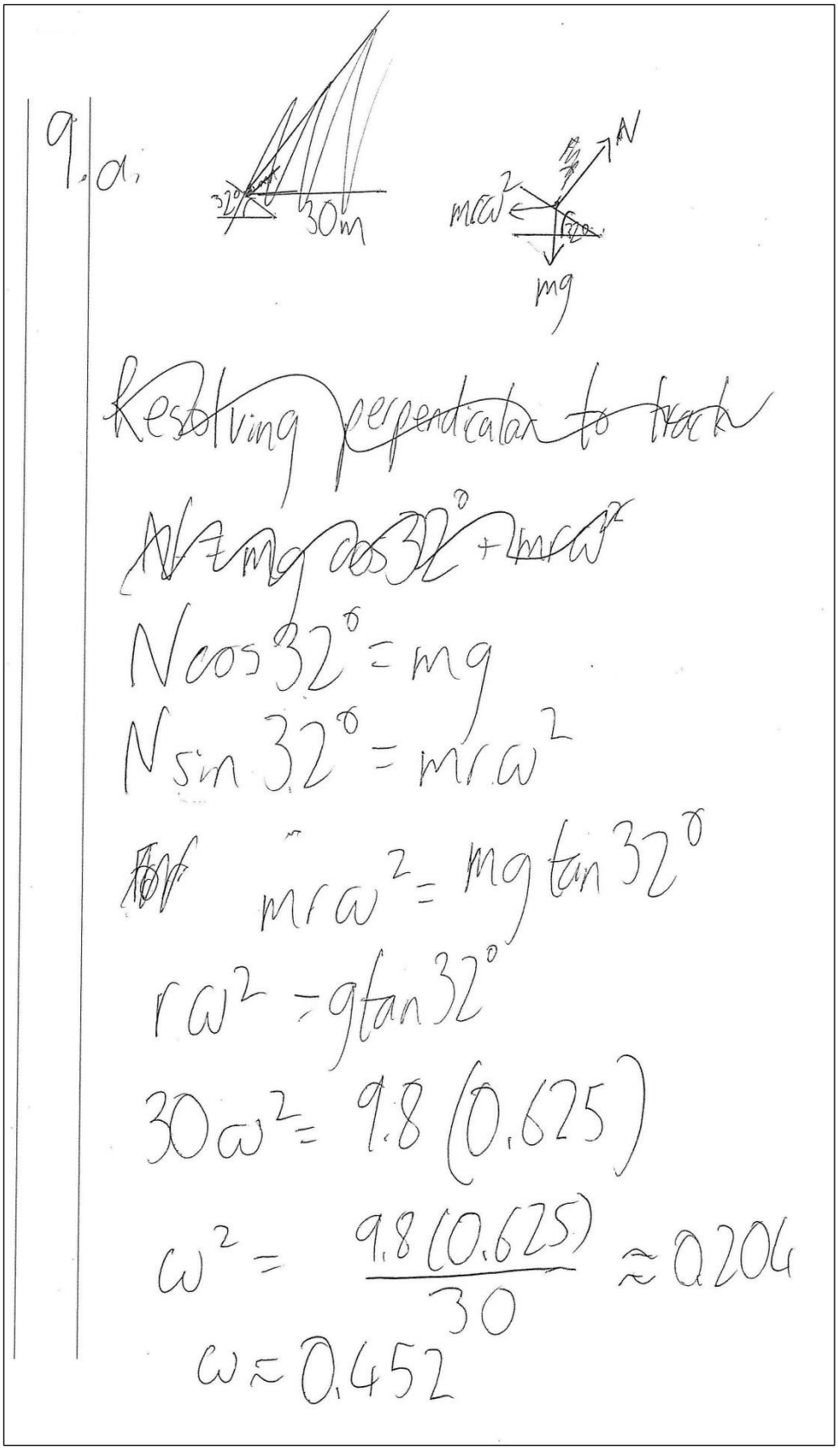
$0.8 = -4A + 4B$

$-16A = 0.1$

$A = \frac{1}{160}$

$y = \frac{1}{160}e^{-20x} + \frac{31}{160}e^{-4x}$

Question 9 (a)
Response 5



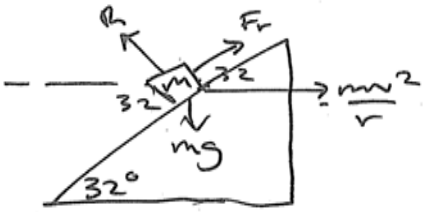
$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.452} \approx 13.9s$

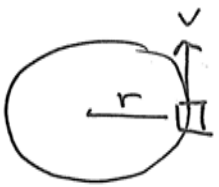
In 300s cyclist makes $\frac{300}{13.9}$

≈ 21.6 laps

21 full laps

Response 6





$r = 30\,m$

$\mu = 0.3$

~~$\frac{mv^2}{r} + mg\sin\theta =$~~

$F_r \quad F_{//} = mg\sin\theta - F_r + \frac{mv^2}{r}\cos\theta = 0$

~~$F_r = \mu R$~~

$F_r = \mu R$

~~$R = \frac{mv^2}{r}\sin\theta$~~

$R = mg\cos\theta - \frac{mv^2}{r}\sin\theta$

(2)

$mg\sin\theta - \mu\left(mg\cos\theta - \frac{mv^2}{r}\sin\theta\right) + \frac{mv^2}{r}\cos\theta = 0$

$\frac{mv^2}{r}\cos\theta + \mu\frac{mv^2}{r}\sin\theta = \mu mg\cos\theta - mg\sin\theta$

$\frac{mv^2}{r}(\cos\theta + \mu\sin\theta) = \ldots$

$v^2 = \frac{r(\mu mg\cos\theta - mg\sin\theta)}{\cos\theta + \mu\sin\theta}$

$= \frac{rg(\mu\cos\theta - \sin\theta)}{\cos\theta + \mu\sin\theta}$

$= 80.4$

$v = 8.96\ldots$

$= 8.97\,ms^{-1}$

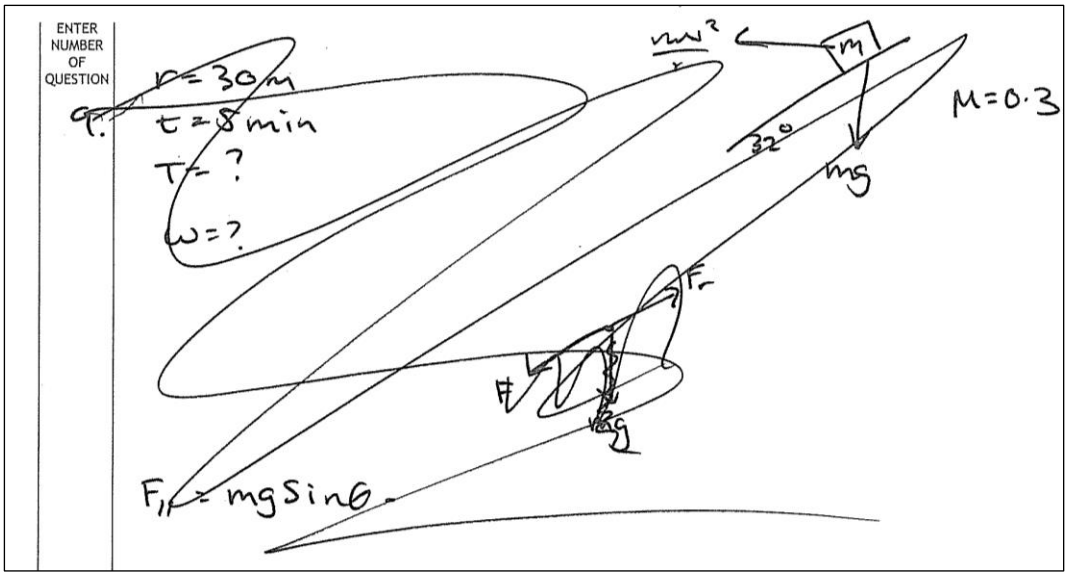
~~$= 8.57\ldots$~~ ~~$v = 9.4\,ms^{-1}$~~

$T = \frac{2\pi}{\omega}$

$\omega = \frac{v}{r} = \frac{8.97}{30} = 0.3\,rads^{-1}$

$T = 21 \qquad f = \frac{1}{T} = 0.047\ldots$

$Laps = 0.047\ldots \times 60 \times 5 = 14.3\ laps$


ENTER NUMBER OF QUESTION
r = 30m
t = 8 min
T = ?
ω = ?
mω²
m
M = 0·3
32°
mg
F
R
mg
F = mg Sin θ

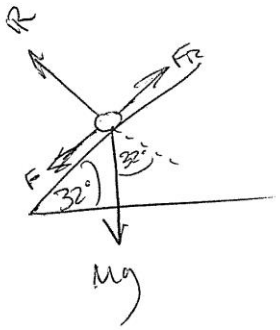
Response 7

ENTER NUMBER OF QUESTION

a a) $r = 30M$

$\theta = 32°$

$\mu = 0.3$



$C = 2\pi r$

$C = 60\pi M$

a.) $t = 5 \times 60 = 300s$

$F_R = \mu R$

$F_R = 0.3R$

$R = Mg\cos 32°$

$F_R = 0.3(Mg\cos 32°)$

$F = \frac{Mv^2}{r} = F_R = (Mg\cos 32°)0.3$

$Mv^2 = 9(Mg\cos 32°)$

$v = \sqrt{9 \times 9.8 \times \cos 32°}$

ENTER NUMBER OF QUESTION

DO NOT WRITE IN THIS MARGIN

$V = 8.64857\ldots$

$\therefore V = 8.65\ ms^{-1}$

~~[illegible]~~

$60\pi s = 8.65 \times 300 = 2595$

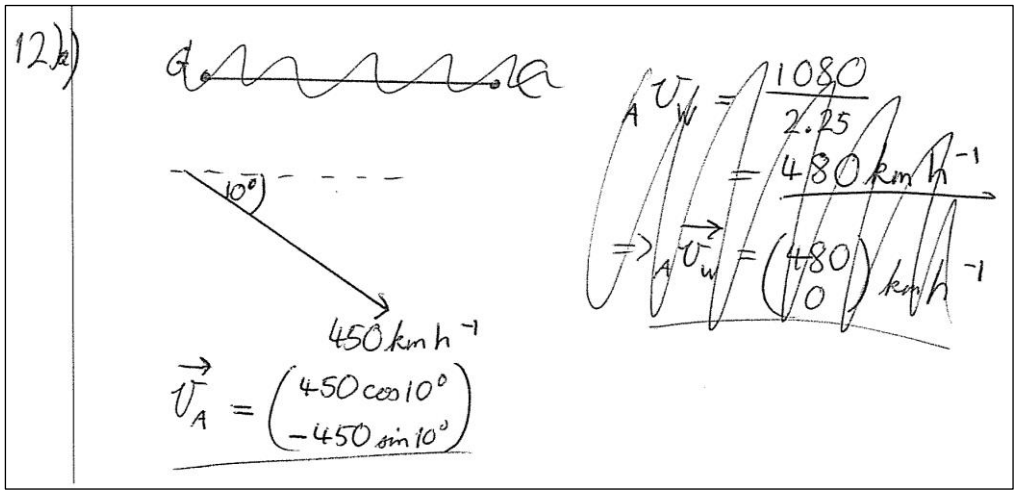
as $C = 60\pi \quad (2\pi r)$

Hence $s = \text{laps} = \frac{2595}{60\pi} = 13.7669\ldots$

so, the cyclist completed 13 full laps

Question 12

Response 8



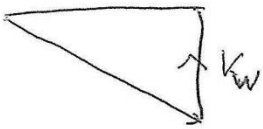
QUESTION

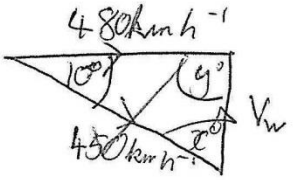
12) a)

$$\vec{v}_w = \vec{v}_A - \vec{v}_w$$

$$\vec{v}_w = \begin{pmatrix} 450\cos 10° \\ -450\sin 10° \end{pmatrix} - \begin{pmatrix} -480 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 450\cos 10° - 480 \\ -450\sin 10° \end{pmatrix}$$





$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$= 480^2 + 450^2 - 2 \cdot 480 \cdot 450 \cdot \cos 10°$$

$$\Rightarrow a \approx 86.4 \text{ km h}^{-1} \Rightarrow |V_w| = 86.4 \text{ km h}^{-1}$$

$$\frac{86.4}{\sin 10°} = \frac{480}{\sin x°} \Rightarrow \sin x° = \frac{480\sin 10°}{86.4}$$

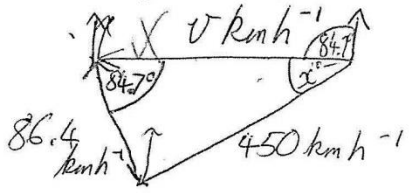
$$\Rightarrow x \approx 74.7°$$

$$\Rightarrow y° = 180 - 10 - 74.7$$

$$= 95.3°$$

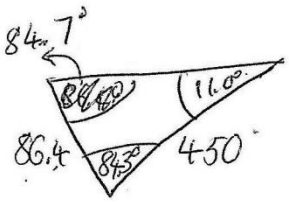
$\Rightarrow$ velocity of wind = 86.4 km h^{-1} on a bearing of 355°.

b) i) Return journey is longer because aircraft is slower



b) ii)

$$\frac{\sin x^\circ}{86.4} = \frac{\sin 84.7^\circ}{450}$$



$$\Rightarrow x = 11.0^\circ$$

$$\Rightarrow \frac{V}{\sin 84.3^\circ} = \frac{86.4}{\sin 11.0^\circ}$$

$$\Rightarrow V = 451 \text{ km h}^{-1}$$

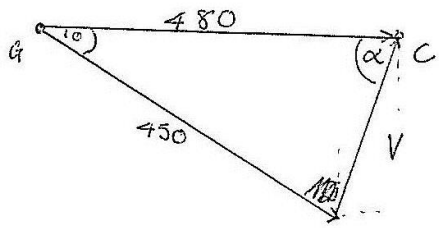
$$\Rightarrow t = \frac{1080}{451} = 2.39 \text{ hours}$$

$\Rightarrow$ journey is longer by ≈ 0.145 hours

$\approx 18.68 = 9$ minutes

Response 9

12a



~~$s = 1080$~~ $u = \frac{1080}{2.25}$

~~$= 2430$ kmh^{-1}~~

$= 480$ kmh^{-1}

~~$a^2 = b^2 + c^2 - 2bc\cos A$~~

~~$V^2 = 480^2 + 450^2 - 2 \times 480 \times 450 \times \cos$~~

$a^2 = b^2 + c^2 - 2bc\cos A$

$V_w^2 = 480^2 + 450^2 - 2 \times 480 \times 450 \times \cos 10$

$= 7463.1$

$V_w = 86.39$ ~~mph~~ kmh^{-1}

$\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{86.39}{\sin 10} = \frac{450}{\sin \alpha}$

$\alpha = \sin^{-1}\left(\frac{450 \times \sin 10}{86.39}\right)$

$= 64.46°$

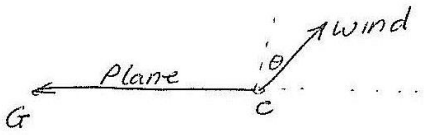
$90 - 64.46°$

$= 25.24°$

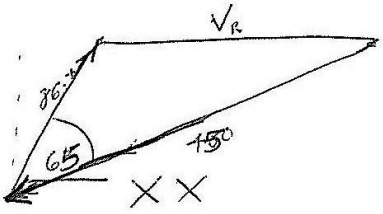
Wind is 86.4kmh^{-1} on a bearing of

025°

bi The ~~cos~~ sinθ component of the wind acts against the plane.



ii



$a^2 = b^2 + c^2 - 2bc\cos A$

$V_R^2 = 150^2 + 86.4^2 - 2\times 150 \times 86.4 \times \cos 65$

$= 147102.2$

$V_R = 420.84 \text{ km h}^{-1}$

$t = \frac{s}{u}$

$= \frac{1080}{420.84}$

$= 2.566 \text{ hrs}$

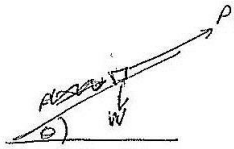
$= 153.98 \text{ minutes}$

$154 - 135 = 19$ minutes longer

Question 14

Response 10

QUESTION 14



resolving perpendicular

$R = W\cos\theta$

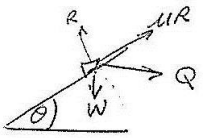
resolving parallel

$P + \mu R = W\sin\theta$

$P + \mu W\cos\theta = W\sin\theta$

~~$P = W\sin\theta - \mu\cos\theta$~~

$\mu = \dfrac{W\sin\theta - P}{W\cos\theta}$



resolving perpendicular

$R = W\cos\theta + Q\sin\theta$

resolving parallel

$\mu R + Q\cos\theta = W\sin\theta$

$\mu(W\cos\theta + Q\sin\theta) + Q\cos\theta = W\sin\theta$

$\mu = \dfrac{W\sin\theta - Q\cos\theta}{W\cos\theta + Q\sin\theta}$

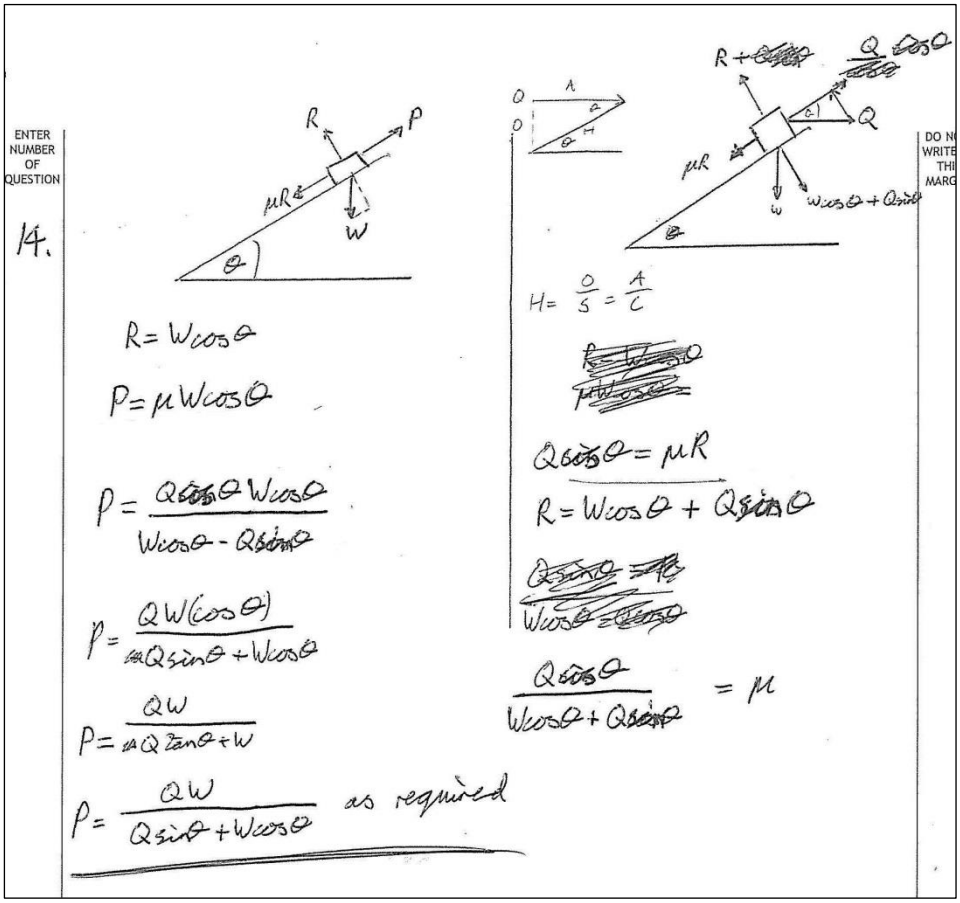
$\dfrac{W\sin\theta - P}{W\cos\theta} = \dfrac{W\sin\theta - Q\cos\theta}{W\cos\theta + Q\sin\theta}$

$W\sin\theta - P = \dfrac{W^2\sin\theta\cos\theta - QW\sin\theta\cos\theta}{W\cos\theta + Q\sin\theta}$

$$-P = \frac{W^2 \sin\theta\cos\theta - QW\sin\theta\cos\theta - W\sin\theta}{W\cos\theta + Q\sin\theta}$$

$$P = \frac{\sin 2\theta(W^2 + QW) + W\sin\theta}{W\cos\theta + Q\sin\theta}$$

Response 11



ENTER NUMBER OF QUESTION

14.

$R = W\cos\theta$

$P = \mu W\cos\theta$

$P = \dfrac{Q\cos\theta\, W\cos\theta}{W\cos\theta - Q\sin\theta}$

$P = \dfrac{QW(\cos\theta)}{\mu Q\sin\theta + W\cos\theta}$

$P = \dfrac{QW}{\mu Q\tan\theta + W}$

$P = \dfrac{QW}{Q\sin\theta + W\cos\theta}$ as required

$H = \frac{O}{S} = \frac{A}{C}$

$Q\cos\theta = \mu R$

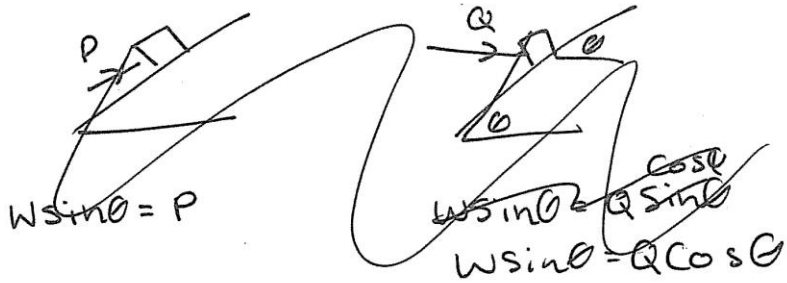
$R = W\cos\theta + Q\sin\theta$

$\dfrac{Q\cos\theta}{W\cos\theta + Q\sin\theta} = \mu$

DO NOT WRITE THIS MARG

Response 12

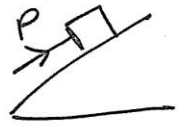
ENTER NUMBER OF QUESTION 14.



$W\sin\theta = P$

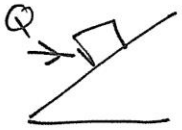
$W\sin\theta = Q\cos\theta$

$W\sin\theta = Q\cos\theta$



$W\sin\theta = P + \mu R$

$W\sin\theta = P + \mu W\cos\theta$ $\quad \mu = \dfrac{W\sin\theta - P}{W\cos\theta}$



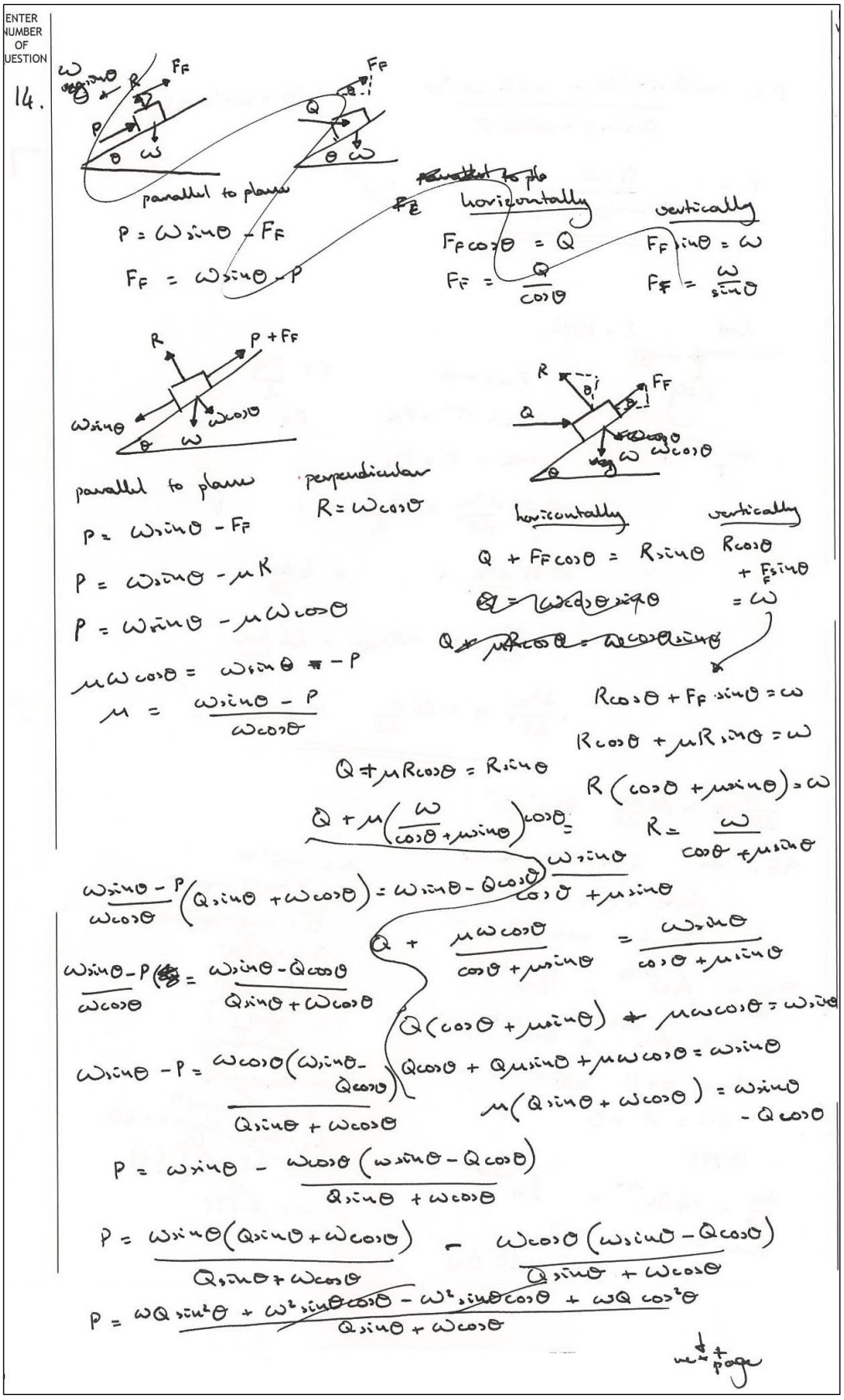
$W\sin\theta = Q\cos Q + \mu W\cos\theta$ $\quad \mu = \dfrac{W\sin\theta - Q\cos\theta}{W\cos\theta}$

P=1

$$\frac{W\sin\theta - P}{W\cos\theta} = \frac{W\sin\theta - Q\cos\theta}{W\cos\theta}$$

$$\frac{W\sin\theta}{W\cos\theta} = \frac{W\sin\theta - Q\cos\theta + P}{W\cos\theta}$$

Response 13


ENTER NUMBER OF QUESTION
14.
parallel to plane
$P = w\sin\theta - F_F$
$F_F = w\sin\theta - P$
horizontally
$F_F\cos\theta = Q$
$F_F = \frac{Q}{\cos\theta}$
vertically
$F_F\sin\theta = w$
$F_F = \frac{w}{\sin\theta}$
parallel to plane
$P = w\sin\theta - F_F$
$P = w\sin\theta - \mu R$
$P = w\sin\theta - \mu w\cos\theta$
$\mu w\cos\theta = w\sin\theta - P$
$\mu = \frac{w\sin\theta - P}{w\cos\theta}$
perpendicular
$R = w\cos\theta$
horizontally
$Q + F_F\cos\theta = R\sin\theta$
vertically
$R\cos\theta + F_F\sin\theta = w$
$Q + \mu R\cos\theta = R\sin\theta$
$R\cos\theta + \mu R\sin\theta = w$
$R(\cos\theta + \mu\sin\theta) = w$
$R = \frac{w}{\cos\theta + \mu\sin\theta}$
$Q + \mu\left(\frac{w}{\cos\theta + \mu\sin\theta}\right)\cos\theta = \frac{w\sin\theta}{\cos\theta + \mu\sin\theta}$
$\frac{w\sin\theta - P}{w\cos\theta}\left(Q\sin\theta + w\cos\theta\right) = w\sin\theta - Q\cos\theta$
$Q + \frac{\mu w\cos\theta}{\cos\theta + \mu\sin\theta} = \frac{w\sin\theta}{\cos\theta + \mu\sin\theta}$
$\frac{w\sin\theta - P}{w\cos\theta} = \frac{w\sin\theta - Q\cos\theta}{Q\sin\theta + w\cos\theta}$
$Q(\cos\theta + \mu\sin\theta) + \mu w\cos\theta = w\sin\theta$
$w\sin\theta - P = \frac{w\cos\theta(w\sin\theta - Q\cos\theta)}{Q\sin\theta + w\cos\theta}$
$Q\cos\theta + Q\mu\sin\theta + \mu w\cos\theta = w\sin\theta$
$\mu(Q\sin\theta + w\cos\theta) = w\sin\theta - Q\cos\theta$
$P = w\sin\theta - \frac{w\cos\theta(w\sin\theta - Q\cos\theta)}{Q\sin\theta + w\cos\theta}$
$P = \frac{w\sin\theta(Q\sin\theta + w\cos\theta)}{Q\sin\theta + w\cos\theta} - \frac{w\cos\theta(w\sin\theta - Q\cos\theta)}{Q\sin\theta + w\cos\theta}$
$P = \frac{wQ\sin^2\theta + w^2\sin\theta\cos\theta - w^2\sin\theta\cos\theta + wQ\cos^2\theta}{Q\sin\theta + w\cos\theta}$
next page

NUMBER OF QUESTION	
14. cont	$P = \dfrac{\omega Q \sin^2\theta + \omega Q \cos^2\theta}{Q\sin\theta + \omega\cos\theta}$ $\quad \sin^2\theta + \cos^2\theta = 1$ $P = \dfrac{Q\omega}{Q\sin\theta + \omega\cos\theta}$