Workshop 1 - Mathematics of Mechanics (Advanced Higher): Question Paper

Analysing results in context and interpreting solutions on basis of physical factors

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Please note these examples are to illustrate how we might ask a candidate to interpret an answer. The skills needed to complete the first part of the question may be simpler than those tested in a course assessment.

Example 1

The position vector of a boat relative to a fixed origin is given as $\mathbf{r} = (6t - t^2)\mathbf{i} + (3 - t^3)\mathbf{j}$

where the unit vectors i and j are directed east and north respectively, with distance measured in kilometres and time in hours.

(i) Find the speed of the boat when after 3 hours.

Answer: $\mathbf{v} = (6 - 2t)\mathbf{i} + (-3t^2)\mathbf{j}$

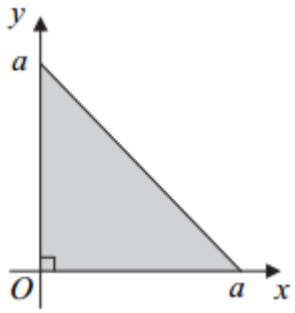
$t = 3 \quad \mathbf{v} = 0 + (-3(3^2)\mathbf{j} = -27\mathbf{j}$

(ii) In which direction is the boat travelling at this time? [1]

Expected answer: SOUTH. We would accept an answer that gave indication of direction eg as a bearing 180°.

Example 2

A uniform triangular lamina is bounded by the line $y = a - x$ and the positive x and y axes, as shown in the diagram.



Explain why the centre of mass of this lamina lies on line $y = x$ [1]

Expected answer: An indication that centre of mass lies on the axis of symmetry – in this case $y = x$

Example 3

Two smooth spheres *A* and *B* of equal radii and masses m_A and m_B respectively lie at rest on a smooth horizontal floor in a line perpendicular to a smooth vertical wall, with *B* closer to the wall. *A* is projected with speed $u\,\text{ms}^{-1}$.

a) Explain why it is not necessary to model the spheres as particles in this question [1]
b) ……..

Expected answer: Because the spheres are congruent/uniform, then centres of mass lie on line parallel to the direction of motion and velocities parallel to this line.

Example 4

Two smooth spheres, P and Q, have equal radii and masses 1 kg and 2kg respectively. They move on a smooth horizontal plane with velocities $\mathbf{v}_P = 2\mathbf{i} + 5\mathbf{j}$ and $\mathbf{v}_Q = -3\mathbf{i} + \mathbf{j}$.

After collision, the velocity of Q is $\mathbf{i} + 2\mathbf{j}$

i) Find the velocity of P after the collision. [Answer: $-6\mathbf{i} + 3\mathbf{j}$]
ii) Find the impulse exerted by B on A. [Answer: $-8\mathbf{i} + 2\mathbf{j}$]

Iii) State a vector in the direction of the line of centres of A and B when they collide.

Expected answer: $-3\mathbf{i} + \mathbf{j}$(or any scalar multiple), recognising that it must be parallel to direction of P after collision.

Example 5

In a Simple Harmonic Motion question……

a) Explain why the speed does not change by equal amounts in equal time intervals.

b) Where there is a resistive force acting, this leads to a second order differential equation of the form $a\frac{d^2x}{dt^2} + b\frac{dx}{dt} + c = 0$.

Candidates need to be able to examine the roots of the auxiliary equation and state if it represents heavy damping (two real distinct roots; critical damping (equal roots) or light damping (complex roots). No further explanation would be needed.

c) Show understanding that, in vertical SHM, the amplitude will determine if all motion is SHM or if body will move freely under gravity at times.

Example 6

A particle moves in a straight line with an acceleration $a = -4v^2$, where $v\,\text{ms}^{-1}$ is its speed after *t* seconds. It has an initial speed - $20\,\text{ms}^{-1}$.

a) Find an expression for its displacement from its starting point.

[Answer: $\frac{1}{4}\ln\left(\frac{20}{v}\right)$]

b) Explain why this model could not be used to find the distance travelled before the particle came to rest. [1]

Expected answer: This expression cannot be evaluated when $v = 0$