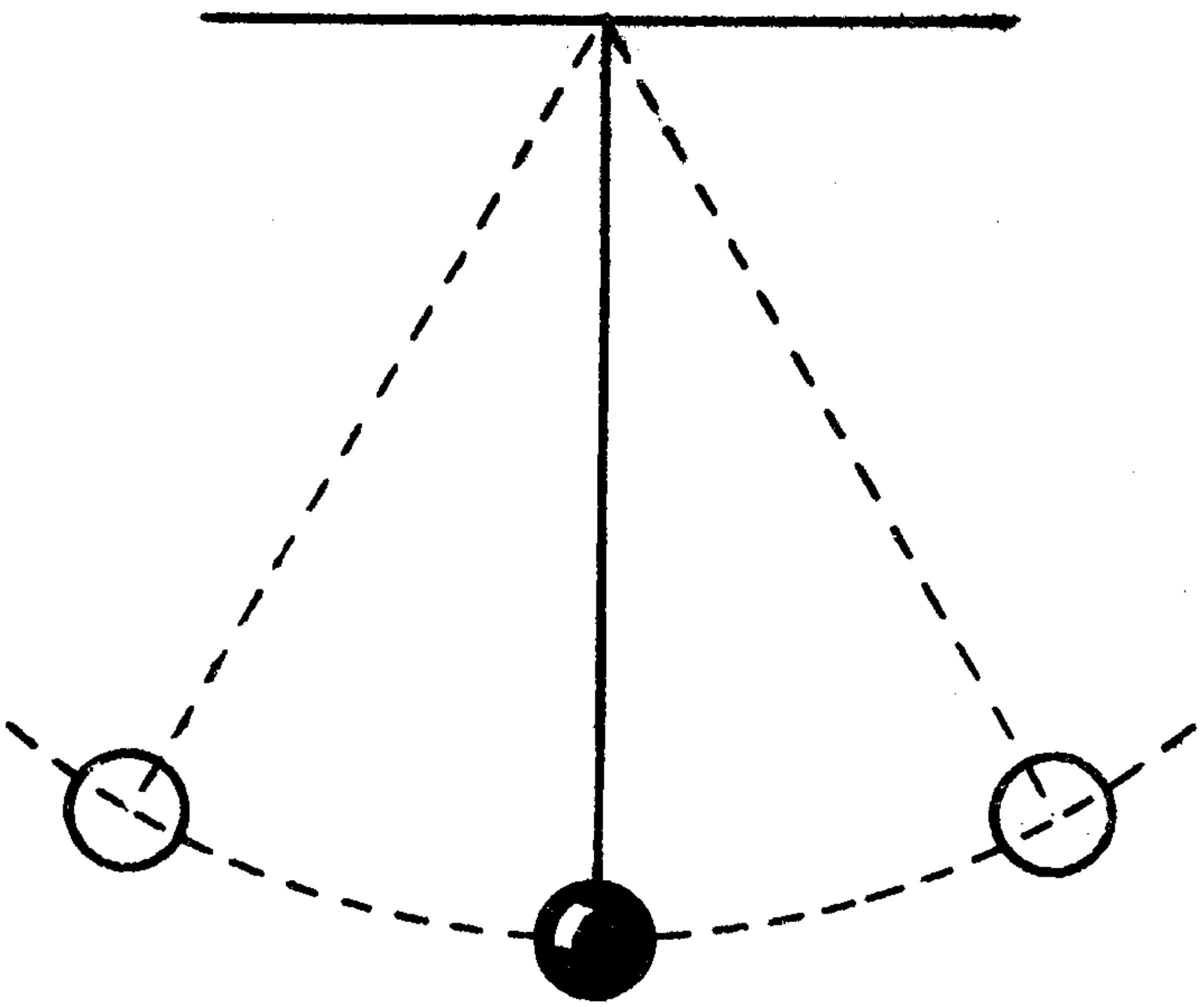
Candidate 2 evidence

Physics Project - Pendulums

To measure the constant g using conventional pendulums



Abstract
Throughout these experiments the ultimate goal was to find a value for g using multiple different pendulums and methods. The first experiment involved using a Simple Pendulum proved to be the most accurate giving a value for g as $9.86 \pm 0.11 ms^{-2}$. For the second experiment a value of $11.1 \pm 1.7 ms^{-2}$ was found using a Compound Pendulum. This experiment suffered greatly from a lack of data points but ultimately the value found was accurate given the circumstances of the experiment. Finally, using a Bifilar Pendulum an accurate result of $10.00 \pm 0.26 ms^{-2}$

CONTENT

Overall underlying Physics
As a rule, every pendulum that is going to be described can be said to be undergoing simple harmonic motion[1]. We understand that every object that is vibrating in this way has a restoring force being acting upon it. The restoring force that is present in all the experiments present are caused by its weight. This means that displacement of weight through each pendulum will affect the oscillation accordingly. Fundamentally, this is the difference between pendulums.

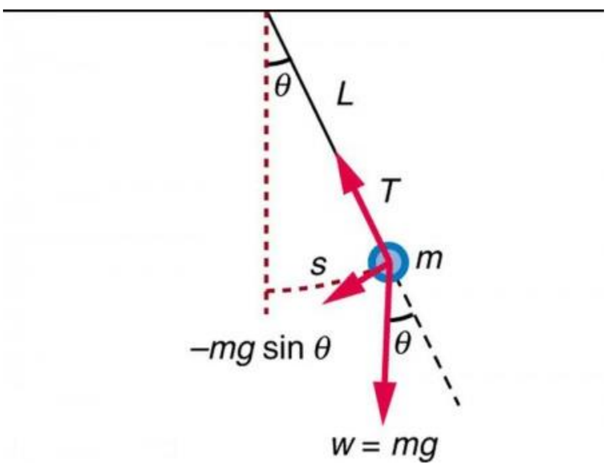
Simple pendulums are defined so that, to all practical purposes, it is a single point mass vibrating. This happens when rest of the pendulums body can be considered to have effectively no weight or negligible weight compared to the "bob" or point mass[2]. Compound pendulums are so that the weight is evenly distributed along a rod and the hinge is so that there is a counterbalance effect. This means that when the hinge point is adjusted to centre of mass there is no simple harmonic motion effect[3]. Finally, the bifilar pendulum is also a rod but it is suspended by two strings and made to oscillate on the x plane and not the y plane as the others were. In this experiment the two strings can be brought closer or further apart to affect the radius of the pendulum [4].

If a pendulum is set in motion so that is swings back and forth, its motion will be periodic and this time frame for a single swing we label with T and measure in seconds. Similarly, a pendulum can be described as having a frequency which is essentially the number of swings this pendulum would produce in a minute. This value can be easily acquired using the formula T = l/f and vice versa f = 1/T

Experiment 1 Simple Pendulum

Aim
To use a simple Pendulum to measure the constant of "g"

Underlying Physics
A simple Pendulum in its simplest form can be described as a point mass suspended by a massless string from a hinge point which doesn't move or change during the swing. When a simple pendulum is displaced from its equilibrium position, there will be a restoring force that moves the pendulum back towards its equilibrium position. This restoring force is caused by its weight which is affected by gravity. [5]



As shown in the diagram the restoring force of the swinging ball I caused by mgsinθ and this can further be demonstrated using some basic trigonometry as shown:

$$sin\,\theta = \frac{0}{H}$$

$$sin\,\theta = \frac{F}{mg}$$

$$F = mg\,sin\,\theta$$

Geometrically, the arc length, s, is directly proportional to the magnitude of the central angle, θ, according to the formula s = rθ. In this diagram the radius of the circle, r, is equal to L, the length of the pendulum. Thus, s = Lθ, where θ must be measured in radians. Substituting into the equation for simple harmonic motion, we get

F = - ks

Now, if we substitute s for Lθ and the restoring force for the equation previously obtained we get

mg sinθ = - k(Lθ)

At this point we solve for the "spring constant" or k for a pendulum and it changes to

mg sinθ = k(Lθ)

$$k = \frac{m\,sin\,\theta}{L\theta}$$

Now it is important to realise that the value of sin θ approximates the value of θ for small angles in radians. Therefore, we can conclude that

$$k \approx \frac{mg}{L}$$

At this point we need to introduce the simple harmonic motion equation for the period of an oscillating system. This equation is derived from the time period of oscillation of a spring. It is independent of gravity. Instead it only depends on the mass of the system and the spring constant. The formula is

$$T = 2\pi\sqrt{\frac{m}{k}}$$

At this point all that is left is to substitute the k equation obtained previously

$$T = 2\pi\sqrt{\frac{m}{mg/L}}$$

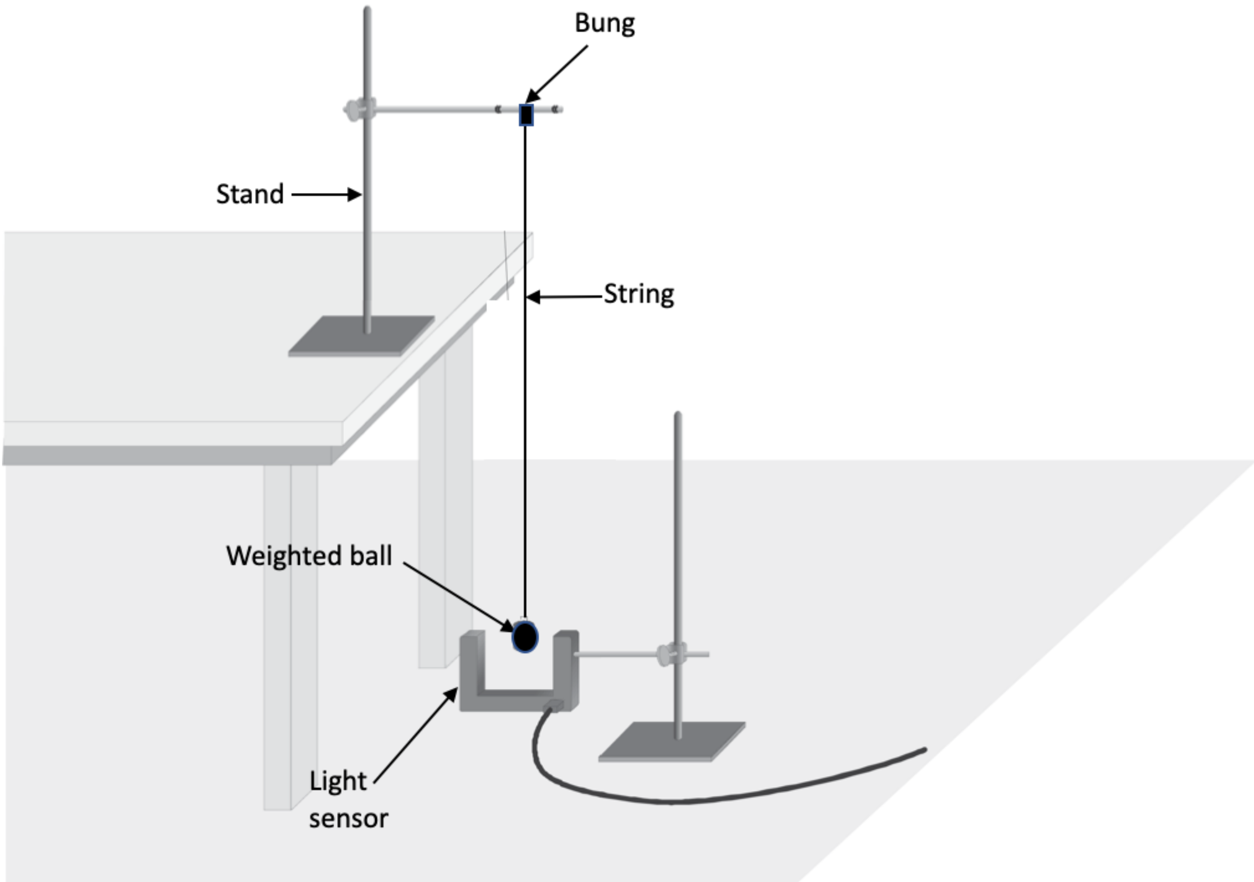
$$T = 2\pi\sqrt{\frac{L}{g}}$$

From here it's a simple rearrangement for g

$$g = 4\pi^2 \left(\frac{L}{T^2}\right)$$

Apparatus
Weighted Ball
String (light)
Bung with slit for string
Clamp
Clamp stand
Pasco Capstone light sensor

Method



A weighted ball is attached to the end of a relatively light string that can easily be lengthened and shortened. The string is threaded through a bung so that the hinge point during the swing never changes. A light sensor is placed behind the ball so that the ball separates a light source from the sensor (this can also be done with a motion sensor). From this point the period of each swing can be measured at different lengths of swing. Repetitions of the found period are taken for different lengths of string so to be later averaged and squared. This information is then used to produce a graph to calculate a gradient.

Data

Experiment 1 Penduilum data						
Length (m)	Average swing times (s)					
	Trial #1	Trial #2	Trial #3	Trial #4	Trail #5	Avg of trials
0.3	11.27	11.27	11.3	11.27	11.27	11.276
0.4	12.95	13	12.95	12.98	12.98	12.972
0.5	14.4	14.4	14.35	14.4	14.35	14.38
0.6	15.75	15.75	15.75	15.75	15.75	15.75
0.7	16.95	16.95	16.95	16.95	16.975	16.955

Calculation examples

Simple Average by Adding up values and dividing by total number of data points
For example

$$\frac{11.27 + 11.27 + 11.27 + 11.3 + 11.27}{5} = 11.276$$

The values for the length of string and the squared period of the swing need to be obtained, before we can calculate the value for g using the formula:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

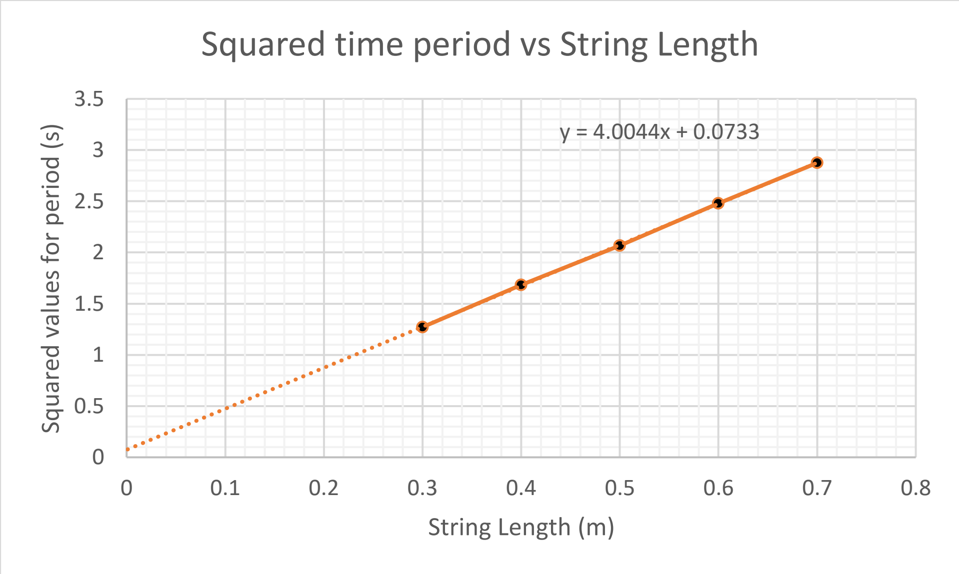
$$T^2 = 4\pi^2 \cdot \frac{L}{g}$$

$$g = 4\pi^2 \cdot \frac{L}{T^2}$$

And so, this data was calculated:

String lengt	Avg. Swin	Avg. singl	squared values	String length (m)	squared value (s)
0.3	11.276	1.1276	1.271482	0.3	1.27148176
0.4	12.972	1.2972	1.682728	0.4	1.68272784
0.5	14.38	1.438	2.067844	0.5	2.067844
0.6	15.75	1.575	2.480625	0.6	2.480625
0.7	16.955	1.6955	2.87472	0.7	2.87472025

This means that we can set a trendline of L against T squared and use that gradient to find the value for g as shown:



Therefore, with this data we can calculate the value for g:

$$m = \frac{4\pi^2}{g}$$

$$\frac{1}{4.0044} \cdot 4\pi^2 = g$$

$$g = 9.858759765$$

$$g = 9.86ms^{-2}$$

Uncertainties

Since the g value depends entirely upon gradient then we can conclude that the gradient uncertainty accounts for the total uncertainty given the relatively tiny scalar and reading uncertainties. The gradient uncertainty was calculated as follows:

Linest function gave:

m	c
4.004374	0.073293
0.022506	0.011695

The uncertainty in the gradient is the uncertainty in g. This gave a percentage uncertainty of:

$$\%m = \frac{0.022506}{4.004374} \times 100 = 0.6\%$$

From here the percentage uncertainty of the gradient could be calculated

This uncertainty can be taken as the total uncertainty in g.

Therefore, uncertainty is
$9.86 \cdot 0.006 = 0.06$

Conclusion

In relation to the hypothesis first set the value obtained is 9.86$\pm$0.06 ms^{-2}

Evaluation

The results obtained by this experiment bear remarkable accuracy and precision to the given value for g. Although, perhaps a more precise measuring implement should have been used compared to a wooden rule – such as a caliper. Also, in this experiment I made no attempt to regulate the arc length of the swing- although this should make no difference to the period in theory, it may have yielded more regular results to control this aspect of the experiment so that the radian approximation makes less of a difference. Secondly, there was a systematic uncertainty which, later realized, was caused due to measuring the length of the string instead of measuring to the center of the point mass. Only measuring the string length meant the line didn't pass through the origin in the graph produce. To solve this problem, you should instead measure from the hinge of the swing to the central point mass of the weight at the end of the string. Finally, taking the average of 20 swings instead of 10 would have yielded a more reliable value for the average swing period.

On the positive side the experiment was repeated many times making the results more reliable as well as reducing any uncertainties. Additionally, the Pasco Capstone light sensor gave 20 readings every second which allowed for a precise value for the period.

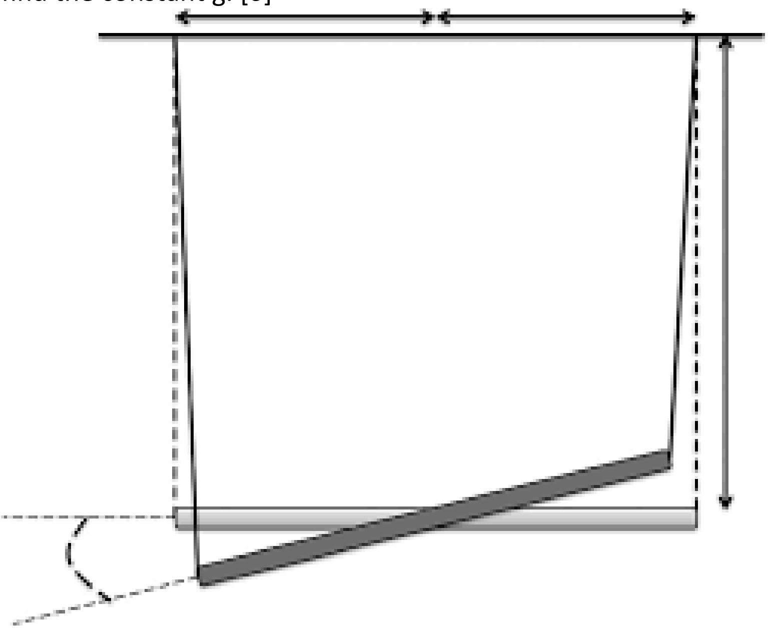
Experiment 2 – Bifilar Pendulum

Aim

To use a Bifilar Pendulum to measure the constant g

Underlying Physics

A Bifilar pendulum is defined as the suspension of an object by two wires and allowed to move freely in the x-axis. This is best displayed using a rod with wires of negligible weight compared to the rod. The classic experiment with this pendulum is used to find the rods inertia but it can also be used to find the constant g. [6]



Firstly, what is noticed about this pendulum is that the tension of the string can be seen to be the restoring force of the pendulum. It can described as follows:

$$T = \frac{mg}{2}$$

This is because the weight is spread between the two wires suspending the rod. From this point we can establish what the greater tension of the string would be when displaced from the equilibrium. This is done using basic trigonometry:

Restoring force = $T \sin\theta = \frac{mg \sin\theta}{2\cos\theta} = \frac{mg \tan\theta}{2}$

Since the restoring force acts upon both sides of the pendulum (for both strings) then the total restoring force is:

$$F = mg\, tan\, \theta$$

Since $tan\theta \approx \theta$ at small angles, then this can be further simplified. Secondly the force is made negative to describe the direction:

$$F = -mg\, \theta$$

Now, since the horizontal displacement of the can be describing with respect to the radius and theta as shown:

$s = r\theta$
Or,
$y = h\theta$

$$\theta = \frac{y}{h}$$

Where y is the horizontal displacement and h is the height of the string suspending the pendulum. Then at this point we can describe theta in this way with respect to the restoring force previously derived.

$$F = -mg\frac{y}{h}$$

This equation can now be taken and considered with torque. Since,

$$\tau = Fr$$

Then,

$$\tau = -\frac{mgyr}{h}$$

At this point we need to consider other equations simultaneously. Both:

$\tau = I\alpha$
And,
$\alpha = \frac{a}{r}$

Which when combined give:
$\tau = \frac{Ia}{r}$
And therefore,

$$\frac{Ia}{r} = -\frac{mgyr}{h}$$

$$a = -\frac{mgr^2y}{Ih}$$

For objects undergoing simple harmonic motion, as this pendulum is, then this equation hold true:

$$\alpha = -\omega^2 y$$

Where omega can be considered in terms of frequency then,

$$\alpha = -\frac{4\pi^2}{T^2} \cdot y$$

Therefore, when this is set this against the restoring force as before we. When we do this we find:

$$\frac{4\pi^2}{T^2} = \frac{mgr^2}{Ih}$$

$$\frac{T^2}{4\pi^2} = \frac{Ih}{mgr^2}$$

The moment of inertia that we would expect from a rod about its center would be:

$$I = \frac{1}{12}ml^2$$

And we can sub this into the equation to find:

$$\frac{T^2}{4\pi^2} = \frac{\frac{1}{12}ml^2h}{mgr^2}$$

$$T^2 = \frac{\pi^2 l^2 h}{3gr^2}$$

At this point we can graph $T^2\ against\ \frac{1}{r^2}\ as\ shown$:

$$T^2 = \frac{\pi^2 l^2 h}{3g} \times \frac{1}{r^2}$$

$$y = mx$$

At this point we would notice that:

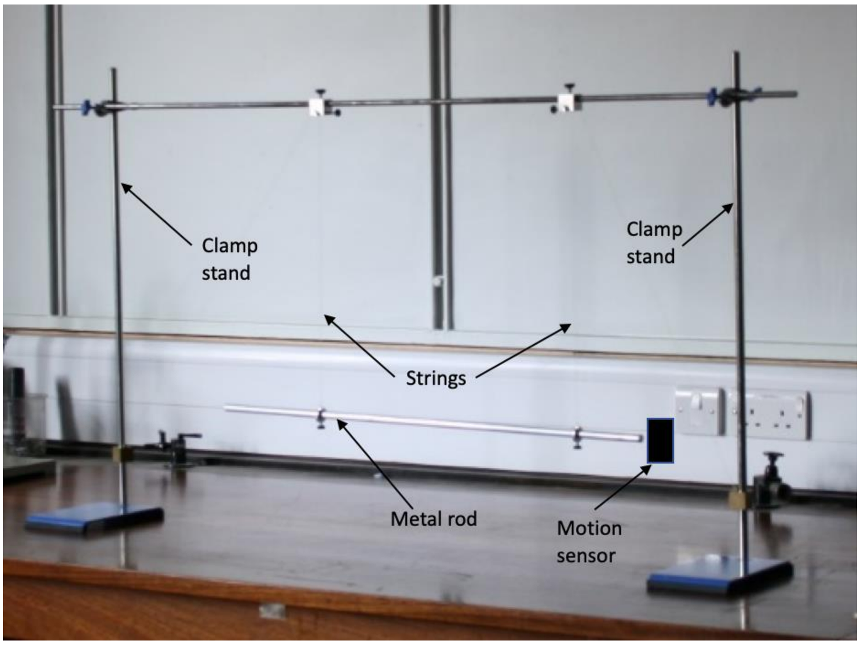
$$m = \frac{\pi^2 l^2 h}{3g}$$

And so, with a simple rearrangement we can find a value for g as shown here:

$$g = \frac{\pi^2 l^2 h}{3 \times gradient}$$

Apparatus
Light sensor
Rod (metal)
Light source (torch)
String
Long Metal Ruler

Method



A metal rod was first measured, and the midpoint found and marked. This midpoint was then used to mark lines at every 5cm intervals starting at a 10cm radius. Then the rod was hung by strings that we both tight but easily loosened and adjusted. In this way, the string was placed at the first mark on the rod at 10cm. A light or motion senor was placed at the end of the pendulum so that it could track the swings as the pendulum moved. A torch was used to make this easier.
The pendulum was then made to swing with a small arc length and the total time was taken for 10 oscillations of the pendulum. This is then repeated 5 times to obtain an average. The strings are then moved to the next position as marked on the pendulum and the whole process is repeated for all points on the pendulum.

Finally, using the information, gathered the time for one swing is calculated and then that value is squared. This result is then plotted against 1 over the radius squared so that the gradient can be used in the equation previously derived.

Data

Radius (m)		Period of Swing					Averges		
		Trial #1	Trial #2	Trial #3	Trial #4	Trial #5	Avg.	Avg. peric	Avg. period squared
0.1		34.2	34.2	34.2	34	34.2	34.16	3.416	11.669056
0.15		22.9	22.8	22.8	23	22.9	22.88	2.288	5.234944
0.2		16.8	17	16.7	16.8	16.7	16.8	1.68	2.8224
0.25		13.6	13.4	13.4	13.6	13.4	13.48	1.348	1.817104
0.3		11.2	11	11.2	11.2	11.2	11.16	1.116	1.245456

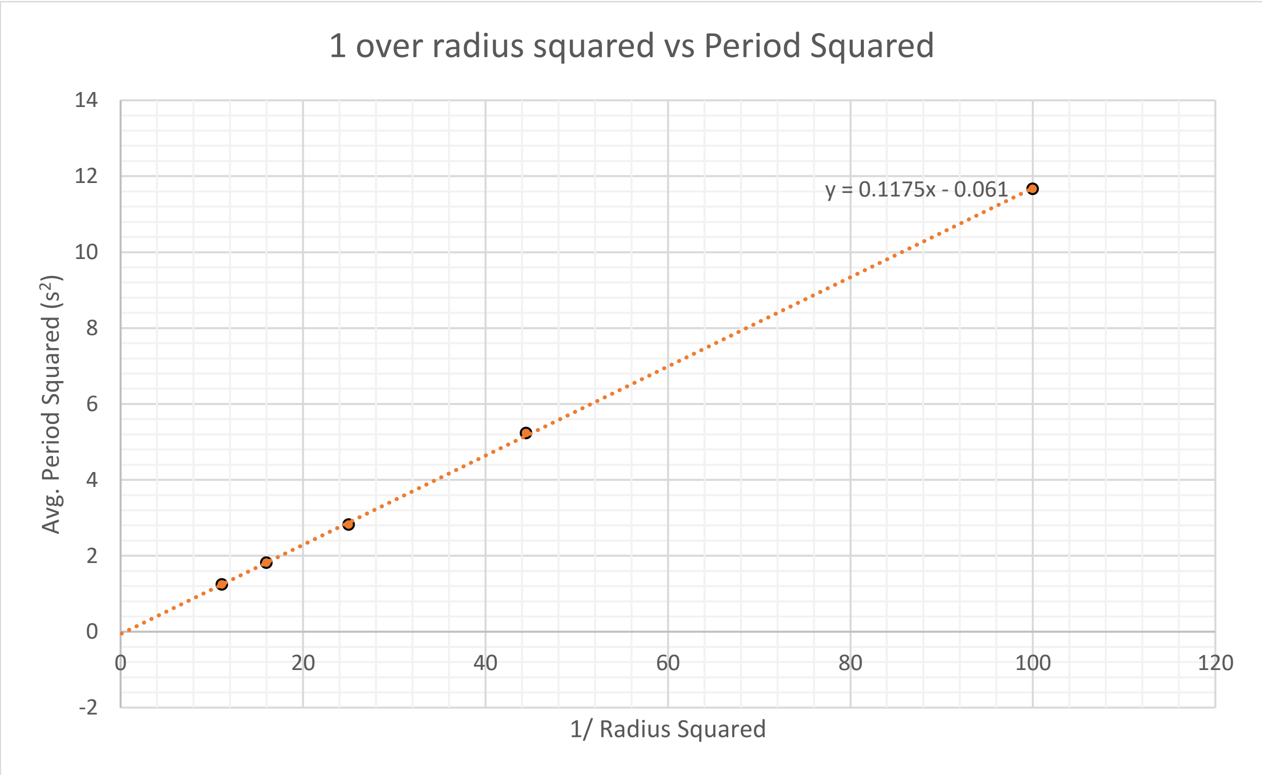
Also recorded both the height of the string suspending the rod and the length of the rod:

Length of rod (cm)	height of string (cm)
75	63.5

Using this data, I set the period squared against one over the radius as shown:

1/ Radius squared	Avg. period squared
100	11.669056
44.44444444	5.234944
25	2.8224
16	1.817104
1.11E+01	1.245456

Graph produced by data:



Calculations

Since,

$$T^2 = \frac{1}{r^2} \cdot \frac{\pi^2 l^2 h}{3g}$$

And we are producing a trendline of,

$$y = xm$$

Then,

$$g = \frac{\pi^2 l^2 h}{3 \cdot gradient}$$

$$g = \frac{\pi^2 \cdot 0.75^2 \cdot 0.635}{3 \cdot 0.1175}$$

g =10.00084914

$g = 10.00\ ms^{-2}$

Uncertainty
For this experiment the gradient uncertainty must be considered along with the reading uncertainties because they cannot be considered as part of the gradient uncertainty as they were in the previous experiments.

m	c
0.117493	-0.06097
0.000746	0.037994

From this point the uncertainty in the gradient can be calculated.

$$\%m = \frac{0.000746}{0.117493} \times 100 = 0.6\%$$

Total percentage uncertainty can be can now be calculated when assuming the reading uncertainty was 1% for both the height and length of the rod. This is done using the Pythagorean equation as follows:
%uncertainty for g

=$\sqrt{\%gradient\ uncertainty^2 + \%length\ uncertainty^2 + \%height\ uncertainty^2}$

%uncertainty for g =$\sqrt{0.6^2 + (1 \times 2)^2 + 1^2}$

%uncertainty for g = 2.3%

This means that the total uncertainty $\pm 0.23ms^{-2}$

Conclusion
With respect to the aim set, the value of g found using a Bifilar Pendulum was 10.00$\pm$0.23ms^{-2}

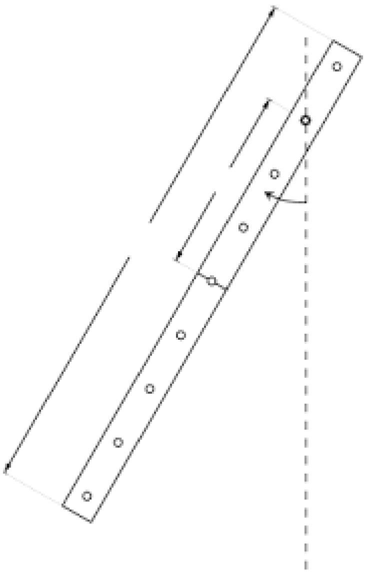
Evaluation
In Evaluation of the results I find them to be both accurate and precise. Although the results do not quite capture the accepted true value for g this was essentially expected given that there was a unavoidable systematic uncertainty that was caused by the rod. The rod itself was inconsistent as at one end had a notable screw fitting while the other end did not. For the purposes of this experiment I ignored this but it weighted side of the rod over the other.
Other than this I took as many reasonable precautions to reduce the uncertainty as possible. For example, I marked on the rod at every 5-centimeter interval before placing on the pendulum- this allowed for much greater accuracy when adjusting the strings.

As before, the Pasco Capstone sensor allowed for a very high level of precision when it came to taking a times for the period because of the high rate of data points collected every second. This helped to reduce the uncertainty and increase precision greatly.
However, during the experiment I had to make several attempts to make the pendulum oscillate properly.

Experiment 3 – Compound Pendulum

Aim
Determine a value for "g" using a compound pendulum

Underlying Physics
When the weight and length of the suspended pendulum are not negligible in comparison with the distance from the axis of suspension to the center of gravity [7]. In other words, when there is a noticeable distance between; the center of gravity and the hinge point, and the hinge point to the end of the pendulum. In this case the pendulum is called a compound pendulum. A rigid body mounted so as to vibrate under the force of gravity is a compound pendulum.
Now, under these conditions, gravity acts as a torque to rotate the whole body of the pendulum. [8]



The restoring torque for an angular displacement θ is:
$\tau = -mg\, l \sin\theta$
Where l is the distance to the centre of mass

As before in the previous experiment we can reduce this since for small amplitudes ($\theta \approx 0$) and so therefore:
$= -mgl\,\theta$

When considering other equations for torque we can conclude that:

$$I\frac{d^2\theta}{dt^2} = -mgl\theta,$$

Where I is the inertia of the body of a pendulum

This equation represents a simple harmonic motion and so the period can be written as

$$T = 2\pi\sqrt{\frac{I}{mgl}}$$

Now, where I_G is the moment of inertia of the body about an axis parallel with axis of oscillation and passing through the centre of gravity G.

$$I_G = mK^2$$

where K is the radius of gyration which the centre of mass travels through

Using this information, we can once again rewrite the equation for the period:

$$T = 2\pi\sqrt{\frac{mK^2+ml^2}{mgl}} = 2\pi\sqrt{\frac{\frac{K^2}{l}+l}{g}}$$

Comparing this to the equation for the period of a simple pendulum, which is:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

We can conclude that

$$L = l + \frac{K^2}{l}$$

By multiplying each term by **l** and rearranging the equation we can reach a simple quadratic formula as shown:

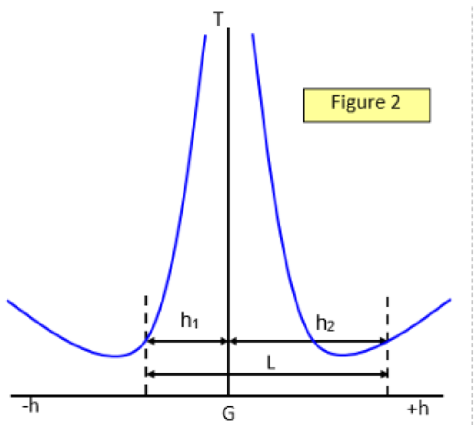
$$l^2 - lL + K^2 = 0$$

Examining this equation, we can start to conclude certain things which we would expect about the nature of this pendulum if we were to be correct. For one, there are two roots for l which will satisfy this equation- this implies that for a given period of a swing there will be two points on the pendulum which are different distances from the end of the pendulum and yet yield the same result for the period of the swing. Since you can consider this pendulum to have effectively infinite number of values for K (because the radius of gyration can be minutely adjusted) you similarly notice that there are an infinite number of these pairings for l values.

Further, given physical necessity both values for l_1 and l_2 must be positive and we can additionally go on to conclude that:

$$l_1 + l_2 = L$$

Ultimately this means that the value for L can be obtained using the method as shown below [9]



And it is this information that I will be obtaining in my experiment to substitute into the equation already used in the first experiment. As shown:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$g = 4\pi^2 \cdot \frac{L}{T^2}$$

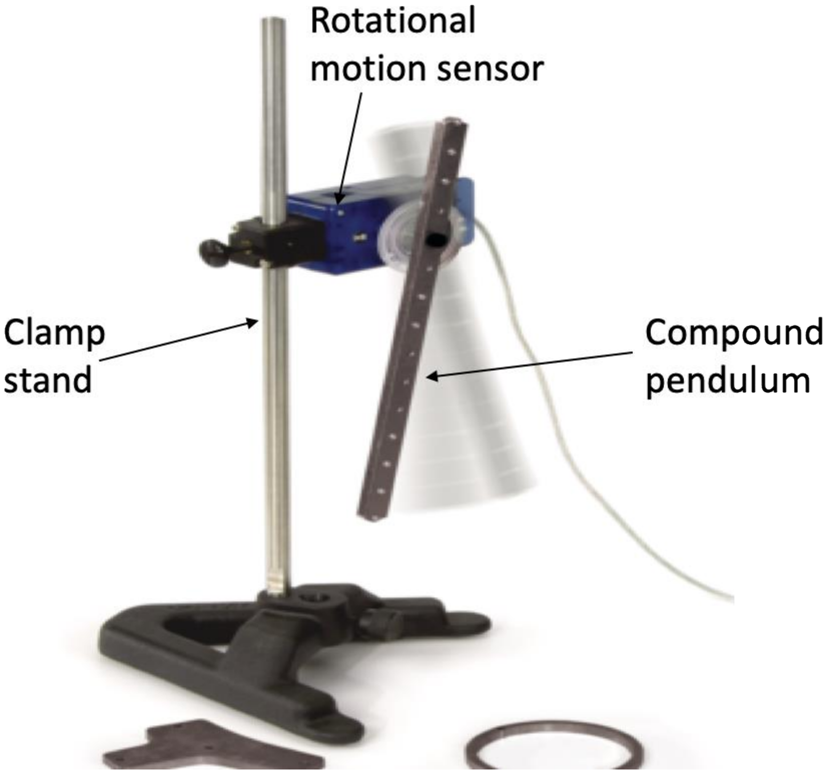
Apparatus

Pasco Physical Pendulum set

Pasco rotational motion sensor

Clamp

Clamp stand



Method

Using apparatus from a pendulum set- as shown in the image above- measurements of the distance from the multiple hinge points to the top of the pendulum were taking by a ruler (this scalar uncertainty can be virtually ignored if the pendulum was manufactured, as ours was). At this point, the pendulum is attached to a Pasco capstone rotational sensor so that the either the rotation or displacement of the pendulum can be measured over time. Measurements for the time of 20 oscillations were taken for increasing hinge points on the pendulum so that the results can be averaged and the period of a single oscillation can be calculated. By using the graph produced by this data, the value for L can be found for each period data point found in the experiment- provided there is a corresponding data on the other side of the graph.

Data

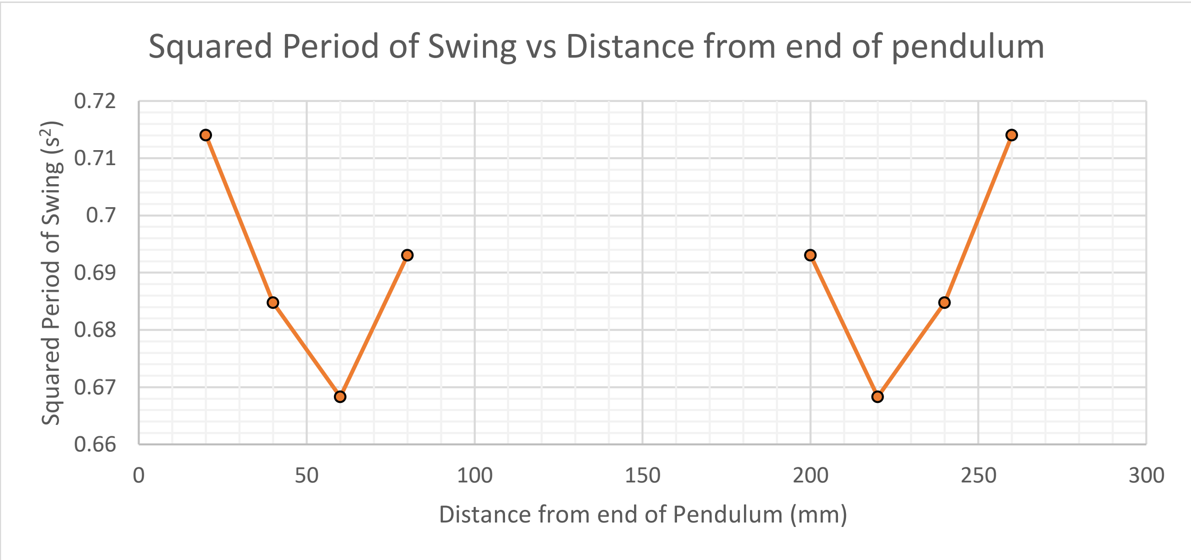
			time for 20 oscillations (s)			average swing time (s	period (s)	period squared
Serial NO (	distance from end(mm)		Trial #1	trial #2	trial#3			
1	20		16.9	16.9	16.9	16.9	0.845	0.714025
2	40		16.55	16.55	16.55	16.55	0.8275	0.68475625
3	60		16.35	16.35	16.35	16.35	0.8175	0.66830625
4	80		16.65	16.65	16.65	16.65	0.8325	0.69305625
1	260		16.9	16.9	16.9	16.9	0.845	0.714025
2	240		16.55	16.55	16.55	16.55	0.8275	0.68475625
3	220		16.35	16.35	16.35	16.35	0.8175	0.66830625
4	200		16.65	16.65	16.65	16.65	0.8325	0.69305625

Calculation example

$$\frac{16.9+16.9+16.9}{3} = 16.9 \qquad 0.845^2 = 0.714025$$

$$\frac{16.9}{20} = 0.845$$

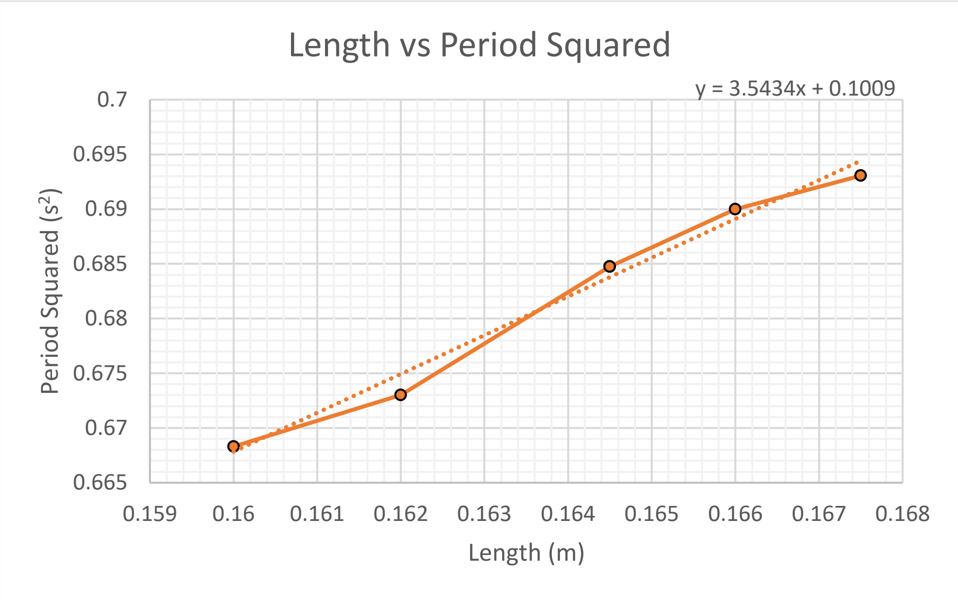
Graph



Values of L obtained by graph against T squared

L (m)	Period squared (s)
0.1675	0.69305625
0.166	0.69
0.1645	0.68475625
0.1645	0.68475625
0.162	0.673
0.16	0.66830625

Graph of L against T squared and trendline



Equations
Using the trendline given we can calculate the value of g as follows:

$$g = \frac{4\pi^2}{m}$$

$$g = \frac{4\pi^2}{3.5434}$$

$$g = 11.141394594 ms^{-2}$$

Uncertainties
While using this method it became apparent that the difference of a single millimeter on the taken value of L had a huge effect on the gradient given by the graph of T squared against L. This is because the difference between the data points along the whole graph was 7.5mm. Essentially the scaled up the reading uncertainty hugely.

The LINEST function was used to get the uncertainty in the gradient:

m	c		
0.277122	-0.025		
0.018786	0.012819		

$$\%m = \frac{0.018786}{0.277122} \times 100 = 6.8\%$$

This gradient uncertainty must also be considered with the reading uncertainty from the graph and the scalar uncertainty from the measurements taken. All of these uncertainties can be combines using the Pythagorean algorithm as follows:

%uncertainty for g=
$\sqrt{\%gradient\ uncertainty^2 + \%reading\ uncertainty^2 + \%scalar\ uncertainty^2}$

$\sqrt{6.8^2 + 5^2 + (1 \times 2)^2}$

= ±8.7%

Therefore, the total uncertainty is found to be ±0.97

Conclusion
With respect to the aim, the calculated value for g using a compound pendulum was found to be $11.1 \pm 0.97ms^{-2}$

Evaluation
The most pressing difficulty encounter during this experiment was the distinct lack of hinge points the were available to use. In fact, ideally, setting up a compound pendulum where the hinge point could be adjusted freely would have the potential to yield far more data points and so attain far more reliable values for L. During this experiment, I was forced to extrapolate two curves from 4 points each – this was by far the most unreliable aspect when obtaining the values for L. If more data points were taken the curve obtained would have been much more reliable.

Secondly, there was a dampening effect that was not calculated. Since the oscillations of the pendulum were seen to come to a stop roughly at the 30 swings mark, we can tell that there was some resistance in the system – this resistance will have had a small effect on the period of the swing. However, the effects of the dampening on the period are only really noticed when there is a more dramatic fall in the peak velocity of the swing. For this reason and for the purposes of this experiment, this effect was ignored.

Finally, there was a potential to take the results from two more hinge points either side of the mid hinge point (which would have yielded no results as, at any position, the pendulum would have been in an equilibrium position). Although ideally theses results would have been taken, due to time pressures this was impossible. More data points would have helped immensely at reducing any uncertainty.

More positively all points that were originally recorded were not only entirely consistent but also perfectly symmetrical. This, I believe, was due to the equipment being used. The Pasco rotational sensor allows for a single device to record the time against the oscillations. This was unlike the other experiments as they required the pendulum set up so to affect the light being received into a sensor. This experiment was respectively very easy to set up. - twenty every second - which was used to calculate the period.

Overall evaluation
As a summery, there was a very consistent accuracy of the experiment which can be attributed to the Low scalar and reading uncertainties throughout the experiment. This was ultimately due to the Pasco Capstone software which offered a very high rate of data points which was more than sufficient for these experiments. The precision of the experiments did, although, vary quite a bit - from the best uncertainty of 1.2% to the worst of over 14%. This was caused by a poor method for the available equipment – there were simple not enough hinge points on the compound pendulum that were necessary for a good reading.

If the experiments were to be redone more care would be taken to keep the arc lengths of all the pendulums both small and consistent. This is because, in every derivation implemented for the experiments, there was a small angle approximation. When measuring the values for the compound pendulum this was not appropriately taken into account and so the results may have been put off because of this.

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