

Candidate 8 evidence

Procedures:

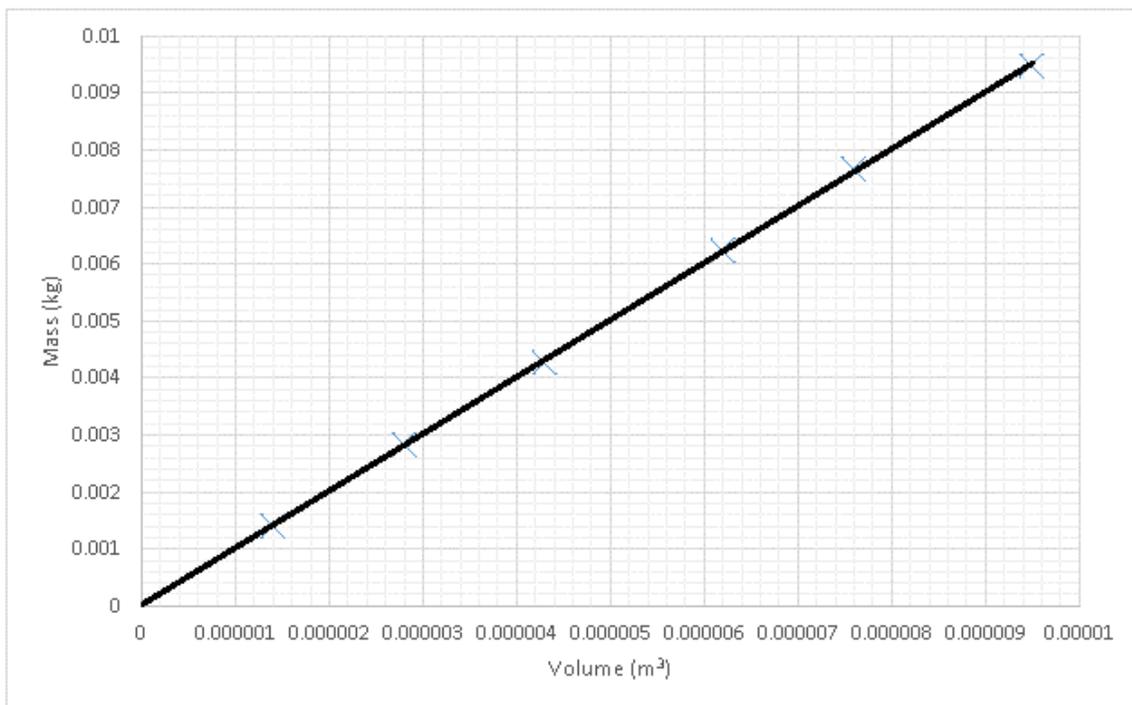
1. Measuring the density of a liquid:

A 10cm³ measuring cylinder was placed on a 2 decimal place balance and the balance was tared. A small volume of liquid was then transferred into the measuring cylinder using a dropper. Readings of both volume and mass were taken. This was repeated until roughly five sets of results were obtained. A graph was plotted of mass (kg) on the y-axis and volume (m³) on the x-axis. The gradient of this graph is the density of the liquid. This procedure was used to calculate the densities of water and glycerol for use in this project.

Results:

1 Density of water:

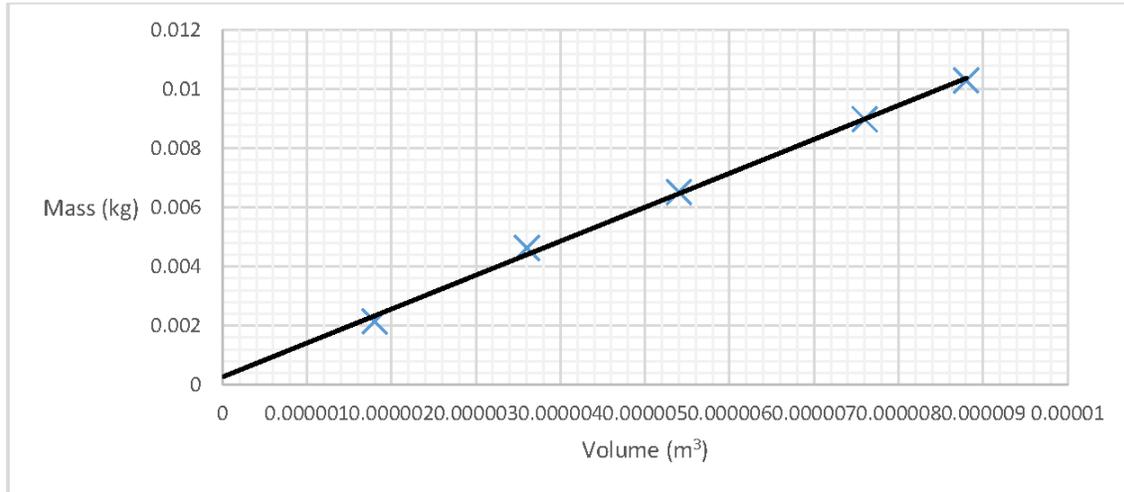
Volume (m ³)	Mass (kg)
0.0000014	0.00141
0.0000028	0.00284
0.0000043	0.00428
0.0000062	0.00624
0.0000076	0.00767
0.0000095	0.00948



$y = 999.87x + 2E-05$ Therefore density of water = 999.87 kg/m³

Density of glycerol:

Volume (m ³)	Mass (kg)
0.0000018	0.00214
0.0000036	0.00462
0.0000054	0.00651
0.0000076	0.00897
0.0000088	0.01030



$y = 1148.5x + 0.0003$ Therefore density of glycerol = 1148.52 kg/m^3

Uncertainty:

LINEST: 1148.514973 0.000260079

 30.84212626 0.00018534

$\Delta m = 30.84 \text{ kg/m}^3$

Density of glycerol = $(1148.52 \pm 30.84) \text{ kg/m}^3$

2 Density of steel spheres:

A two decimal place digital meter was used to measure the radius of different sized spheres. The volume of each sphere was calculated using the formula $V = \frac{4}{3}\pi r^3$. Each sphere was then weighed using a two decimal place balance. The density of the steel spheres was then calculated using the formula $p = m/V$. Five different sizes of steel spheres were used in this project.

Size	Mass (g)	Radius (mm)	Volume (m ³)	Density (kg/m ³)
1	0.88	3.00	1.13×10^{-7}	7781
2	4.07	5.00	5.24×10^{-7}	7773
3	8.98	6.50	1.15×10^{-6}	7806
4	16.70	8.00	2.14×10^{-6}	7787
5	63.65	12.5	8.18×10^{-6}	7780

Example calculation (Size 1):

$$V = \frac{4}{3}\pi r^3$$

$$V = \frac{4}{3} \times 3.14 \times (0.003)^3$$

$$V = 1.13 \times 10^{-7} \text{ m}^3$$

$$p = \frac{m}{V}$$

$$p = 0.00088 / 1.13 \times 10^{-7}$$

$$p = 7788 \text{ kg/m}^3$$

Uncertainties:

Example calculation (size 1):

$$\Delta \text{mass} = 0.5\% \times 0.88 + 0.01 = 0.0144 \quad , \quad \% \Delta \text{mass} = \frac{0.0144}{0.88} \times 100 = 1.64\%$$

$$\Delta r = 0.5\% \times 3.00 + 0.01 = 0.025 \quad \% \Delta r = \frac{0.025}{3.00} \times 100 = 0.83\%$$

$$\% \Delta V = 3 \times \% \Delta r = 2.50\%$$

$$\% \Delta p = \sqrt{(\% \Delta m)^2 + (\% \Delta V)^2}$$

$$\% \Delta p = \sqrt{(1.64)^2 + (2.50)^2}$$

$$\% \Delta p = 2.99\%$$

$$2.99\% \times 7788 = 232 \text{ kg/m}^3$$

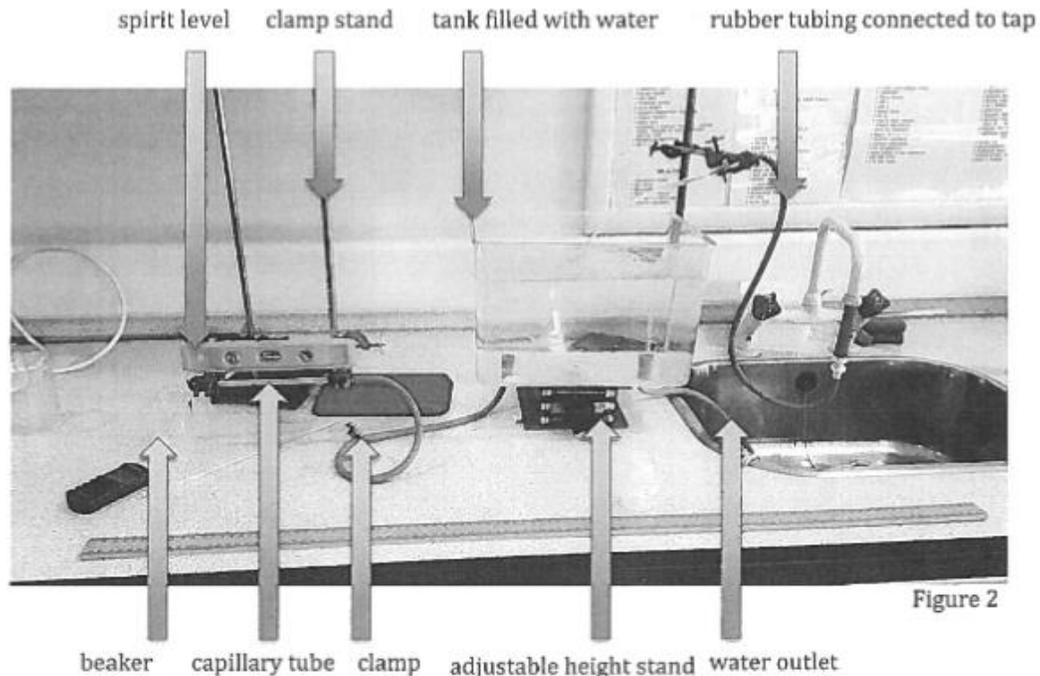
$$\text{Density} = (7788 \pm 232) \text{ kg/m}^3$$

Size	Δmass	$\%\Delta\text{mass}$	Δr	$\%\Delta r$	$\%\Delta V$	$\%\Delta D$	ΔD
1	0.014	1.64	0.025	0.83	2.50	2.99	232
2	0.03	0.75	0.035	0.70	2.1	2.23	173
3	0.055	0.6125	0.0425	0.654	1.962	1.962	153
4	0.094	0.56	0.05	0.625	1.875	1.875	146
5	0.328	0.52	0.0725	0.58	1.74	1.74	135

Note: For sizes 3-5 $\%\Delta V$ was the dominant uncertainty as $\%\Delta V > 3 \times \%\Delta\text{mass}$

3. Poiseuille's Law:

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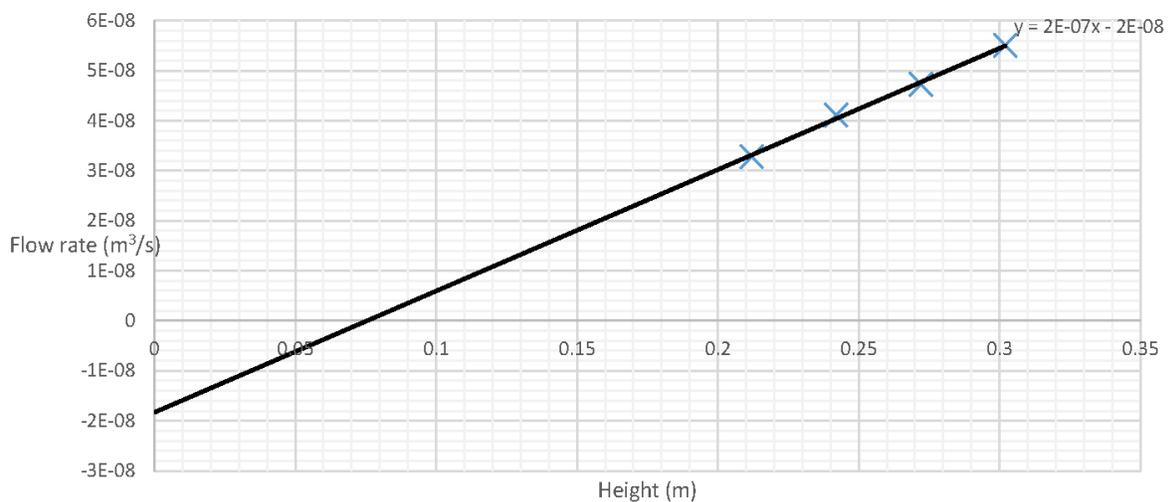


A Vernier microscope was used to measure the radius of the capillary tube. This was done by first measuring the top of the hole, then the bottom and dividing the difference by two. The measurement at the top was 9.46cm, the bottom was 9.53cm, giving a radius of 0.035cm. A wooden meter stick was used to measure the length of the capillary tube and the height of the water level above the level of the capillary tube. The apparatus shown in Figure 2 was set up. A small amount of grease was put at the underside of the end of the capillary tube to ensure the water would drip into the beaker and not run along the tube. The spirit level was used to ensure that the capillary tube and water tank were level. The clamp stands were adjusted until the capillary tube was level. The tap was switched on and the clamp opened. It was important that the water was coming out of the capillary tube in drops so that the flow was not allowed to become turbulent. A time of three minutes was measured using a stopwatch and the volume of water collected in the beaker during this time was measured using a measuring cylinder. This experiment was then repeated, only changing the height of the water level above the level of the capillary tube which affected the volume of water collected. All other variables remained constant. The volumes of water collected for each different height over three minutes were recorded. A graph was then plotted of flow rate (m^3/s) on the y-axis against Height (m) on the x-axis. From the gradient of this graph the viscosity (μ) of the water could be calculated.

Height (m)	Volume (cm ³)
0.332	15.9
0.302	9.9
0.272	8.5
0.242	7.4
0.212	5.9

Note: All of the above volumes were collected over a period of 3 minutes.

In the following graph, the first line from the table was not included.



LINEST:

$$2.42593\text{E-}07 \quad -1.83185\text{E-}08$$

$$9.6225\text{E-}09 \quad 2.49396\text{E-}09$$

$$m = 2.426 \times 10^{-7}$$

$$\frac{\pi r^4 \rho g}{8l\mu} = 2.426 \times 10^{-7}$$

$$\mu = \frac{3.14 \times (3.5 \times 10^{-4})^4 \times 999.87 \times 9.81}{8 \times 0.273 \times 2.426 \times 10^{-7}}$$

$$\mu = 8.72 \times 10^{-4} \text{ Nsm}^{-2}$$

Uncertainties:

$$\Delta m = 9.62 \times 10^{-9}$$

$$\% \Delta m = \frac{9.62 \times 10^{-9}}{2.43 \times 10^{-7}} \times 100 = 3.96\%$$

$$\Delta l = 0.0005 \text{ m}$$

$$\% \Delta l = \frac{0.0005}{0.273} \times 100 = 0.18\%$$

$$\Delta \rho = 5.98 \text{ kg/m}^3$$

$$\% \Delta \rho = \frac{5.98}{999.87} \times 100 = 0.60\%$$

$$\Delta r = 0.000005 \text{ m}$$

$$\% \Delta r = \frac{0.000005}{0.00035} \times 100 = 1.43\%$$

$$\% \Delta r^4 = 4 \times 1.43 = 5.72\%$$

$\% \Delta l$ & $\% \Delta \rho < \frac{1}{3} \% \Delta r^4$ and can therefore be ignored.

$$\% \Delta \mu = \sqrt{(\% \Delta m)^2 + (\% \Delta r^4)^2}$$

$$\% \Delta \mu = \sqrt{(3.96)^2 + (5.72)^2}$$

$$\% \Delta \mu = 6.96\%$$

$$\mu = (8.72 \pm 0.61) \times 10^{-4} \text{ Nsm}^{-2}$$

$$\text{Textbook value: } \mu = 8.91 \times 10^{-4} \text{ Nsm}^{-2}$$

4. Stokes' Law:

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A tall glass tube, which was closed at one end, was clamped vertically and filled with water (4). The spirit level was used to check that the tube was level, this was important as it prevented the spheres from hitting the sides. Figure 3 shows the experimental setup, which was set up near the bottom end of the tube.

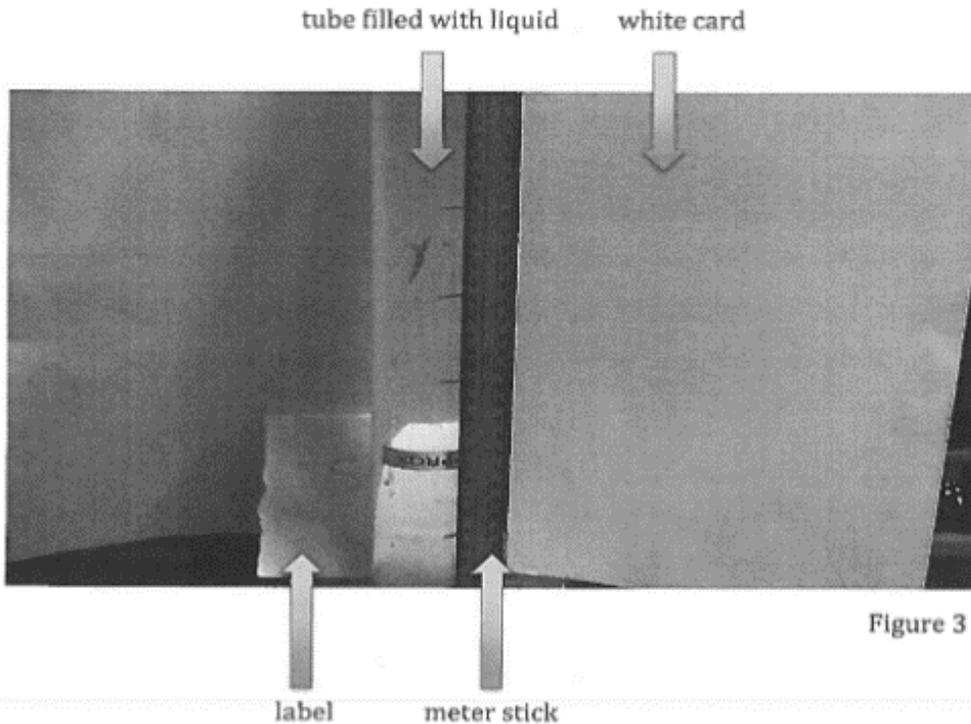


Figure 3

The meter stick was attached to the clamp holding the tube in place using blue-tack. The label indicated which size of sphere was currently being used. A large magnet was used to get the spheres back out of the tube in between experiments. The white card was put in place to make the spheres clearly visible against the white background as they fell down the tube.

4.1 Stokes' Law: Water & Stopwatch:

Size	t ₁ (s)	t ₂ (s)	t ₃ (s)	t ₄ (s)	Mean t (s)
1	0.23	0.26	0.31	0.27	0.2675
2	0.27	0.30	0.30	0.31	0.2950
3	0.25	0.33	0.32	0.31	0.3025
4	0.31	0.37	0.34	0.35	0.3425
5	0.37	0.38	0.34	0.31	0.3500

All times given in the table above are for a height of 0.49 m.

Using $v = d/t$ to calculate the velocities of the spheres:

Size	v (m/s)
1	1.832
2	1.661
3	1.620
4	1.431
5	1.400

Example calculation (size 1)

$$V_t = \frac{2r^2(p_s - p_l)g}{9\mu}$$

$$\mu = \frac{9.81 \times 2(3 \times 10^{-3})(3 \times 10^{-3})(7788 - 999.87)}{9 \times 1.832}$$

$$\mu = 0.073 \text{ Nsm}^{-2}$$

Size	Viscosity (Nsm ⁻²)
1	0.073
2	0.223
3	0.386
4	0.662
5	1.652

All of these values for μ lie so far apart from each other and so far away from the textbook value of $8.91 \times 10^{-4} \text{ Nsm}^{-2}$ (5) that it is clear that there was a major problem with the experimental procedures. It would be pointless to calculate uncertainties for these results, as a large error had been made that supersedes any experimental uncertainties.

4.2 Stokes' Law: Water & Video Camera:

Instead of using a stopwatch to measure the time taken for the spheres to pass through a marked distance, a video camera was used to film the spheres as they fell down the tube. The white card was now placed behind the glass tube so that the spheres would be clearly visible on the videos. These videos could then be analyzed on a computer. The software used to analyze all videos taken for this project was Windows Live Movie Maker. This program allowed for accurate time measurements while the meter stick was used to measure the distance travelled during that time. Individual numbers on the meter stick could not be seen clearly on the videos but each centimeter division was visible so that it was possible to count the number of centimeter divisions that the sphere had passed through. Once again $v = d/t$ was used to calculate the velocity which then allowed for the viscosity to be calculated using Stokes' Equation. Only a couple of results were taken using this method before then switching from water to glycerol.

Only two experimental results were taken in this section. One was for size 3 sphere and one for size 5 sphere.

Size	d (cm)	t (s)	μ (Nsm ⁻²)
3	10	0.06	0.376
5	9	0.07	1.791

These values are still very far apart and not concordant with the textbook value. Uncertainties were not calculated for this section for the same reason stated in results 4.1.

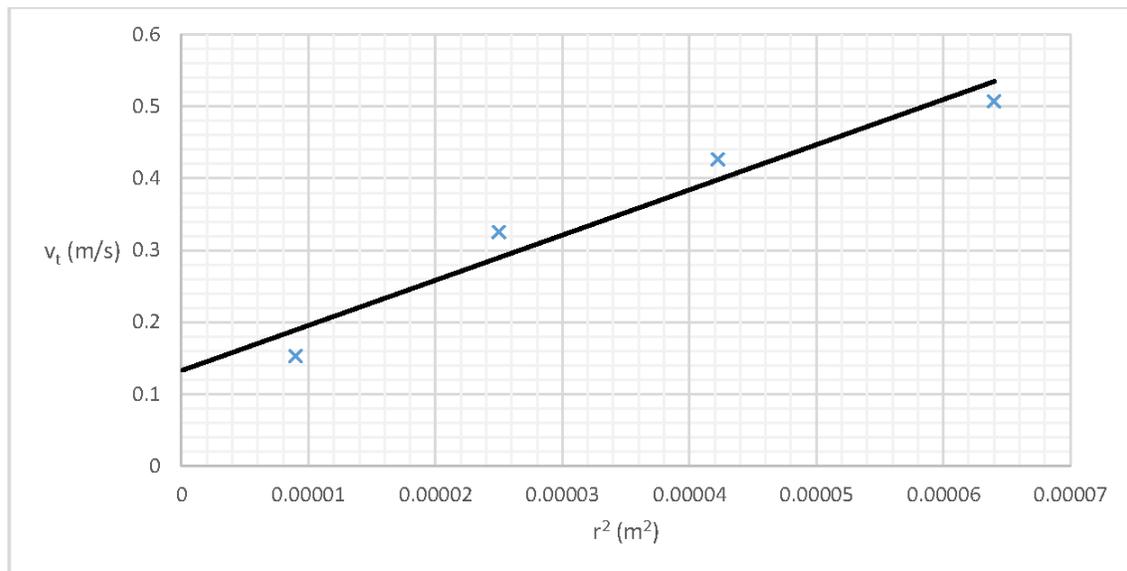
Stokes' Law: Glycerol & Video Camera

The water was emptied out, the tube dried and then filled with glycerol. It was allowed to stand overnight so that the air bubbles could rise and disappear. The same procedure described in 4.2 was carried out; the only difference being that the water in the tube had been replaced by glycerol. Three videos were taken for each size of steel sphere. From the results an average velocity for each size of sphere was calculated. It was assumed that these velocities were terminal. A graph was then plotted of V_t (m/s) on the y-axis and r^2 (m²) on the x-axis. From the gradient of this graph the viscosity (μ) of glycerol could be calculated.

Size	d (cm)	t start (s)	t end (s)	time taken (s)	v_t (m/s)	mean v_t (m/s)
1	11	6.41	7.11	0.70	0.157	
1	11	6.61	7.34	0.73	0.151	(1) 0.153
1	6	3.88	4.28	0.40	0.150	
2	11	3.23	3.56	0.33	0.333	
2	14	2.59	3.03	0.44	0.318	(2) 0.325
2	12	2.48	2.85	0.37	0.324	
3	11	1.4	1.67	0.27	0.407	
3	7	2.49	2.65	0.16	0.438	(3) 0.427
3	10	1.73	1.96	0.23	0.435	
4	10	2.39	2.59	0.20	0.500	

4	12	2.39	2.62	0.23	0.522	(4)	0.507
4	10	1.73	1.93	0.20	0.500		
5	10	2.98	3.18	0.20	0.500		
5	16	1.59	1.85	0.26	0.615	(5)	0.527
5	14	2.76	3.06	0.30	0.467		

A graph was plotted of V_t (m/s) on the y-axis and r^2 (m²) on the x-axis:



LINEST: 6285.453273 0.132540996
 1117.473436 0.045347474

$$m = 6285.45$$

$$\frac{2(p_s - p_l)g}{9\mu} = 6285.45$$

$$\mu = \frac{2(7789.8 - 1148.5) \times 9.81}{9 \times 6285.45}$$

$$\mu = 2.30 \text{ Nsm}^{-2}$$

(an average value of p_s was used in this calculation)

Uncertainties:

$$\Delta m = 1117.47$$

$$\% \Delta m = \frac{1117.47}{6285.45} \times 100 = 17.78\%$$

$$\Delta p_l = 30.84$$

$$\% \Delta p_l = \frac{30.84}{1148.5} \times 100 = 2.69\%$$

$$\% \Delta p_s \text{ ranging from } 1.7\% - 3.0\%$$

Δm is the dominant uncertainty.

$$\mu = (2.30 \pm 0.41) \text{ Nsm}^{-2}$$

Textbook value: $\mu = 0.942 \text{ Nsm}^{-2}$ (5)

Conclusion:

Poiseuille's Law:

$$\underline{\mu = (8.72 \pm 0.61) \times 10^{-4} \text{ Nsm}^{-2}}$$

Textbook value: $\mu = 8.91 \times 10^{-4} \text{ Nsm}^{-2}$ (5)

Stokes' Law:

No satisfactory results were obtained in 4.1 and 4.2 when attempting to find the viscosity of water using Stokes' Law.

Stokes' Law using glycerol instead of water:

$$\underline{\mu = (2.30 \pm 0.41) \text{ Nsm}^{-2}}$$

Textbook value: $\mu = 0.942 \text{ Nsm}^{-2}$ (5)

Discussion & Evaluation:

Planning

I first planned to measure the viscosity of water using a viscometer. The only one that the school had was an Otswald viscometer, but it got broken before I could use it properly. I checked out other viscometers, but they were too expensive to buy. I then researched for other ways of measuring viscosity, and found the Poiseuille method and the Stokes method, which I could do because the school technician could make the equipment I needed.

One difficulty found with the Poiseuille method at the start was that the water came out of the capillary tube with a slow drip rate and the experiment would take too long. To solve this problem I replaced the plastic beaker I used to hold the constant head of water with a bigger tank, which could hold a bigger head of water which made a greater pressure on the water in the capillary and increased the drip rate so I could do the experiment in a reasonable time.

With the Stokes method, I had to use reasonably small ball bearings and even then it was difficult to stop them hitting the sides of the tube. They also fell quite fast and it was difficult to tell if they had reached terminal velocity and even measure their speed. I got round this by videoing the falling spheres and analysing the playback to find their speed, and then by replacing the water in the tube with glycerol, which is more viscous.

Poiseuille's Law

Poiseuille's method was the better method of the two used in this project as the value of $\mu = (8.72 \pm 0.61) \times 10^{-4} \text{ Nsm}^{-2}$ obtained lies very close to the textbook value for the viscosity of water which is $\mu = 8.91 \times 10^{-4} \text{ Nsm}^{-2}$ (5). The percentage uncertainty in this final result is also relatively low at 6.96% compared to a much larger percentage uncertainty in Stokes' Law. These factors suggest good precision and accuracy in the final result.

Obtaining this accurate final result was only possible when ignoring the top line in the table of results for Poiseuille's Law. This point was far off lying on the best-fit line of the graph and was not concordant with any other measurements taken. It is noted that this was the measurement taken at the maximum height of the water level above the level of the capillary tube. The most likely explanation for this measurement not following the pattern is that the height had been increased to such an extent that the pressure became high enough for the flow in the capillary tube to become turbulent. Poiseuille's Law only applies to laminar flow so if the flow in the capillary tube were to become turbulent the experiment would not work. Because there was such a high chance that the flow had become turbulent it was decided not to include this measurement in any of the calculations so as not to spoil the other results which all lay on the best-fit line suggesting good accuracy.

Even though all point on the graph lay on the same best-fit line, the y-intercept was considerably below zero, where it should have been according to the theory. Whilst this does not affect the gradient and therefore the final results, it suggests that there was a systematic error, i.e. all points on the graph were out by the same amount. There are a number of factors that could have caused this error, which occurred throughout the whole experiment. When measuring the radius of the capillary tube using the Vernier Microscope, it was observed that there were small bits of grease stuck inside the capillary tube. These

then opposed the flow through the capillary tube, causing an error throughout all results. Another potential source of error was the stopwatch. There was a chance that the old stopwatch used to measure the time of three minutes was not running accurately anymore, causing an error in the flow rate throughout all results taken. However the systematic error is more likely to have been caused by the bits of grease stuck inside the capillary tube.

Stokes' Law:

At first the aim of this project was to simply calculate the viscosity of water using different experimental methods but this needed to be changed when it came to Stokes' law. The results in 4.1 are so far off the textbook value and each result is so different from the others that it was clear that there was a major problem with this experiment. The reason that this experiment did not work is that for Stokes' law to apply, the spheres must have reached their terminal velocities. In this experiment the spheres were still accelerating which ruins the point of the experimental procedures of measuring their terminal velocities. Another large source of errors in the first experiment was the use of a stopwatch to measure very short time intervals of $<0.5s$. It is almost impossible to accurately measure such short time intervals by hand. To eliminate this error and to prove that the error must lie in the fact that a terminal velocity had not been reached, the stopwatch was replaced by a video camera.

The purpose of repeating the experiment using a video camera to measure the time interval instead of a stopwatch (4.2) was to prove that the spheres had not reached a terminal velocity and that the bad experimental results were not caused by a bad method of measuring time. It was not necessary to take a full set of results here, instead only a couple of measurements were made and values for μ calculated from them. The two values obtained in this section were as before far from the quoted textbook value and each other. Because the error in measuring the time had been minimized, it was now clear that spheres must still have been accelerating even as they approached the bottom end of the tube. A way to try and fix this problem would be to use a taller glass tube, but such equipment was not available. Instead, glycerol was then used instead of water. Glycerol has a higher density than water which meant that the spheres would reach a terminal velocity faster than in water.

A full set of results was taken for glycerol, using a video camera to calculate the terminal velocity of the spheres (4.3). The results from this experiment were the only satisfactory results obtained from Stokes' Law. The final result of $\mu = (2.30 \pm 0.41) \text{ Nsm}^{-2}$ lies reasonably close to the textbook value of $\mu = 0.942 \text{ Nsm}^{-2}$ (5) for the viscosity of glycerol. Unlike in Poiseuille's Law the range of uncertainties does not overlap with the textbook value but it is still within close proximity, suggesting decent accuracy. One source of error using Stokes' Law is that the larger spheres were affected by turbulence when falling through the tube. To prevent this, a tube with a larger diameter could be used but no other one was available for use in this project. The uncertainty in the value of 17.78% is very large and clearly points out that Poiseuille's Law was the more accurate and reliable method to use.

There are various possibilities for further work on this project. For one the effect of temperature on viscosity could be investigated. This would not require any new experimental techniques. Another interesting technique that I did not have time to fully research was Reynold's Number, which look at the transition from laminar to turbulent flow.