Candidate 5 evidence

Abstract

The aim of my investigation is to examine the effect of varying frequency on current in resistive, capacitive and inductive circuits. Three different circuits were set up to investigate this: a purely resistive circuit, a purely capacitive circuit and a purely inductive circuit. A further two circuits were set to determine the resonant frequency in the circuit: a series RLC circuit and a RLC circuit with an inductor and a resistor in parallel.

From these five experiments, I concluded that:

- In a purely resistive circuit, frequency has no effect on current
- In a purely capacitive circuit, the current is directly proportional to the frequency as current increases as frequency increase.
- In a purely inductive circuit, the frequency is inversely proportional to current as current decreases as frequency increases
- In a series RLC circuit, the current increases until the resonance frequency is reached, and then begins to fall. I found the resonant frequency of this circuit to be 400 ± 0.5% Hz
- In a parallel RLC circuit, the current decreases until the resonant frequency is reached, and then begins to rise. I found the resonant frequency in this circuit to be the same as in the series RLC circuit.
- The theoretical resonant frequency was calculated to be 397.9 ± 7% Hz.

General Underlying Physics

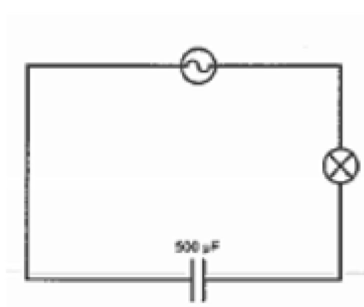
In an ac circuit, the frequency is the number of positive to negative cycles the voltage in the circuit makes every second. This frequency has varying effects on the current in the circuit, depending on what type of circuit is being used.

Experiment 1 – Current and frequency in a Capacitive Circuit

Aim

The aim of this experiment was to investigate what effect varying frequency had on the current on a purely capacitive series circuit.

Underlying Physics



Take a circuit with a 50 Hz power supply, a lightbulb and a 500 µF capacitor as shown in the diagram. In this circuit, the bulb will light up, giving the impression that there is a current flowing through the capacitor. Though this is not the case as no current can flow through a capacitor once the plates are fully charged, i.e. Q = CV, as there is an insulator between the two plates. Instead, the capacitor is being charged and discharged almost simultaneously 50 times every second, as this is the frequency of the supply. It is these currents that are passing through the lamp and lighting it up, making it appear that there is a current flowing through the capacitor, this would also be registered by an ammeter.

If the size of this capacitor was to be decreased, the bulb in the circuit would dim as the charging and discharging currents would decrease. This shows that larger capacitors have a smaller reactance therefore providing less opposition against the flow of charge. This means that increasing the frequency of the supply, while keeping the supply voltage and capacitance in the circuit constant, increases the current in the circuit as I =Q/t and the amount of charge must charge and discharge the capacitor but in a shorter space of time.[1]

This can be proved mathematically by the following:

Take an ac supply voltage V across a capacitor of capacitance C and let its value at time t be given by:

$$V = V_0 \sin \omega t$$

Here, V_0 is the maximum amplitude of the voltage and $\omega = 2\pi f$ where f is the frequency of the supply. The charge Q in the capacitor at time t is given by:

$$Q = CV$$

The current I flowing through this capacitor can be written as:

$$\begin{aligned} I &= \frac{dQ}{dt} \\ &= \frac{d}{dt}(CV) = C\frac{d}{dt}(V_0 \sin \omega t) \\ &= CV_0 \omega \cos \omega t \end{aligned}$$

Thus, the current through the capacitor C, leads the voltage by π/2 radians. We can also write:

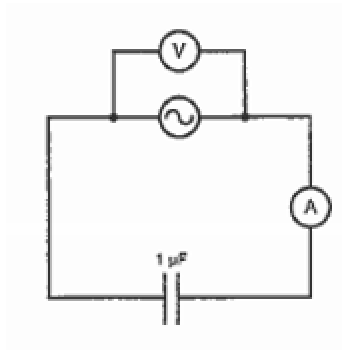
$$I = I_0 \cos \omega t$$

Where I_0 is the maximum amplitude of the current and is equal to $CV_0\omega$, and can be rearranged to give:

$$\frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}} = \frac{1}{\omega C} = \frac{1}{2\pi fC}$$

This value is the capacitive reactance of the capacitor which proves that if the capacitance and voltage remain constant and the frequency is increased, then the current must increase to keep the values equal.[2]

Apparatus



- AC power supply
- Multimeter across power supply used to ensure the voltage across the power supply remains constant as well as accurately measure the frequency of the supply
- Ammeter to measure the current in the circuit
- 1 μF capacitor

Method

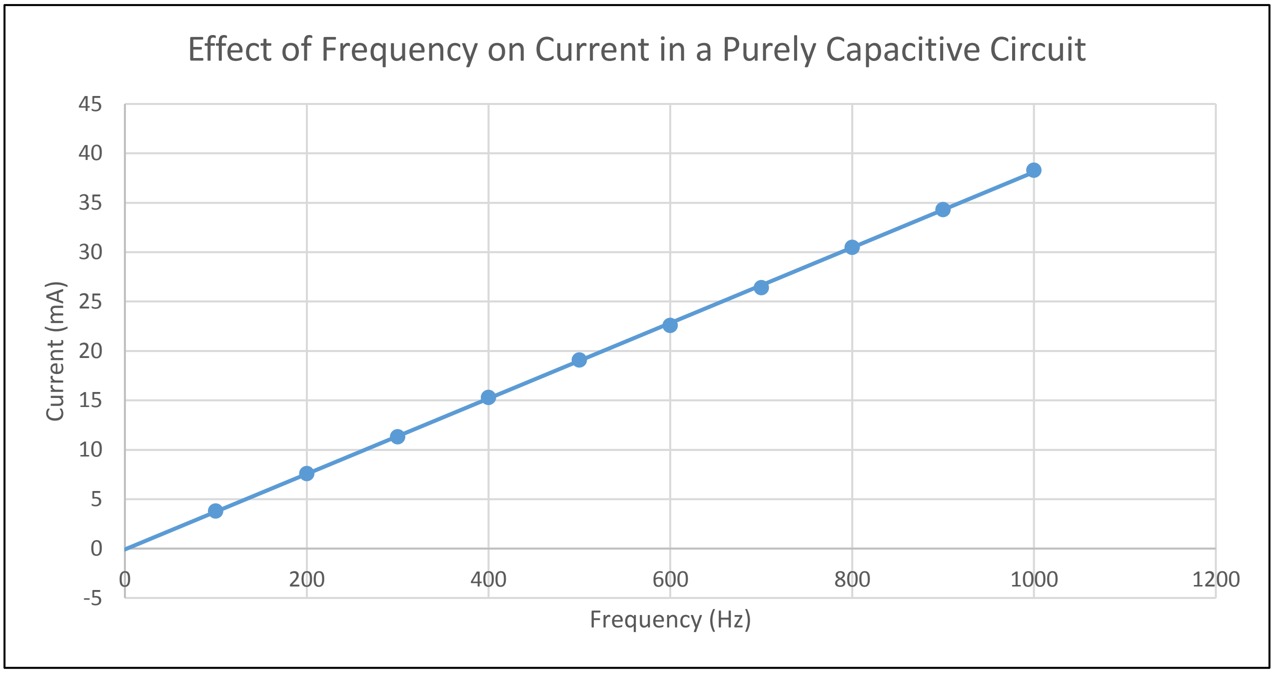
The above circuit was set up as shown in the diagram and the frequency of the supply set to 100 Hz. The supply voltage was 6.1 V and the voltmeter was used to make sure it stayed constant at this voltage throughout the whole experiment. The current was measured on the ammeter and the experiment repeated a further nine times, increasing the frequency in increments of 100 Hz each time and recording the current with each different frequency.

Results

Voltage (V) ± 0.1V	Capacitance (μF) ± 10%	Frequency (Hz) ± 1Hz	Current (mA) ± 0.1mA
6.1	1	100	3.8
6.1	1	200	7.6
6.1	1	300	11.3
6.1	1	400	15.3
6.1	1	500	19.1
6.1	1	600	22.6
6.1	1	700	26.4
6.1	1	800	30.5
6.1	1	900	34.3
6.1	1	1000	38.3

Conclusion

The results of this experiment show that as the frequency of the signal generator is increased, the current increases directly proportional to frequency. Therefore, the graph created, shown below, is straight line which passes through the origin.


Effect of Frequency on Current in a Purely Capacitive Circuit
Current (mA)
45
40
35
30
25
20
15
10
5
0
-5
0
200
400
600
800
1000
1200
Frequency (Hz)

Experiment 2 – Current and Frequency in a Purely Inductive Circuit

Aim

The aim of this experiment was to investigate what effect varying frequency had on the current in a purely inductive circuit.

Underlying Physics

Take a current I flowing through an inductor with inductance L at time t given in the form

$$I = I_0 \sin \omega t$$

Here, I_0 is the maximum amplitude of the current and $\omega = 2\pi f$ where f is the frequency provided from the ac power supply. The equation for the back EMF ε due to the changing current in the circuit is given by:

$$\varepsilon = -L\frac{dI}{dt}$$

And as I = I_0 sinωt:

$$\varepsilon = -L\frac{d}{dt}(I_0 \sin \omega t)$$

Differentiating gives:

$$= -\omega L I_0 \cos \omega t$$

Assuming the inductor provides no resistance, the supply voltage must be equal and opposite to the back EMF to allow the current to flow. Therefore we can write:

$$V = -\varepsilon = \omega L I_0 \cos \omega t$$

Thus, the supply voltage leads the current by π/2. We can write:

$$V = V_0 \cos \omega t$$

Here, V_0 is the maximum amplitude of the voltage, given by:

$$V_0 = \omega L I_0$$

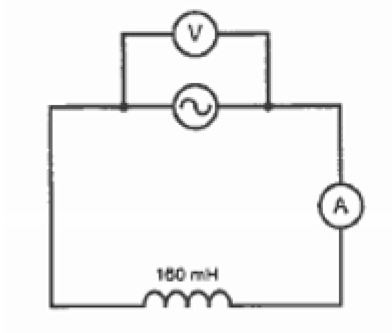
Rearranging gives:

$$\frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}} = \omega L = 2\pi fL$$

This value 2πfL is the inductive reactance X_L of the inductor, measured in ohms.

Therefore , if L and V_{rms} are kept constant and the frequency is increased, I_{rms} must increase to keep the values equal.[3]

Apparatus



- AC power supply
- Multimeter across power supply used to ensure the voltage across the power supply remains constant as well as accurately measure the frequency of the supply
- Ammeter to measure the current in the circuit
- 160 mH inductor

Method

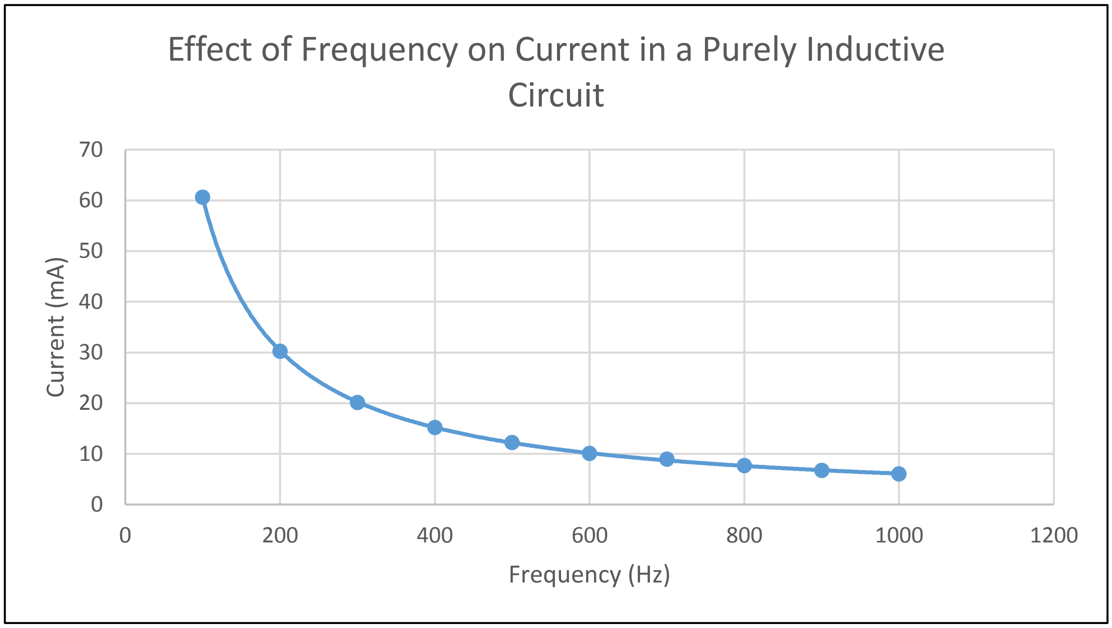
The method for this experiment was identical to the previous experiment. The above circuit was set up as shown in the diagram and the frequency of the supply set to 100 Hz. The supply voltage was 6.1 V and the voltmeter was used to make sure it stayed constant at this voltage throughout the whole experiment. The current was measured on the ammeter and the experiment repeated a further nine times, increasing the frequency in increments of 100 Hz each time and recording the current with each different frequency.

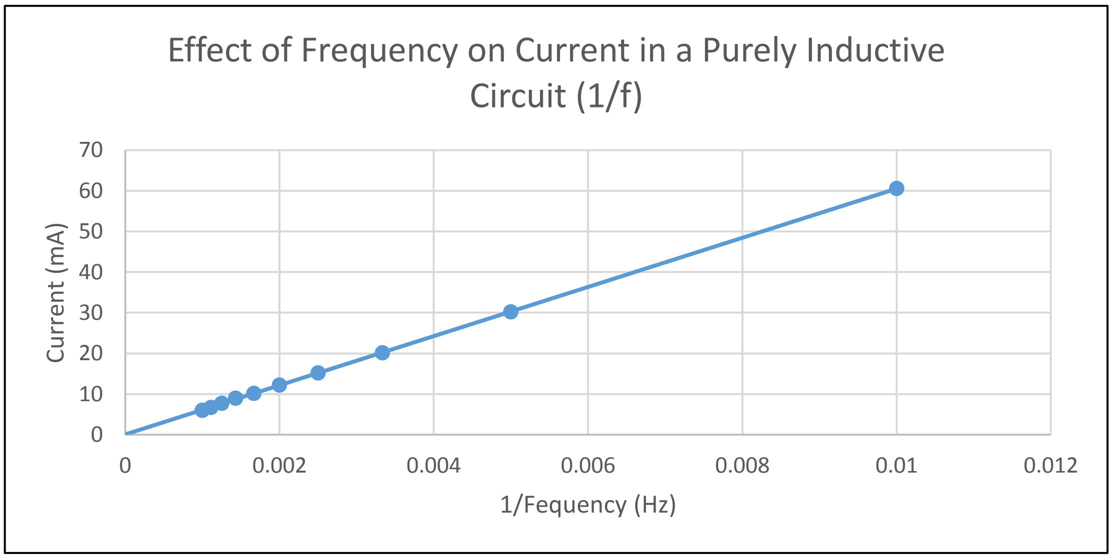
Results

Voltage (V) ± 0.1V	Inductance (mH) ± 10%	Frequency (Hz) ± 1Hz	Current (mA) ± 0.1mA
6.1	160	100	60.6
6.1	160	200	30.2
6.1	160	300	20.1
6.1	160	400	15.2
6.1	160	500	12.2
6.1	160	600	10.1
6.1	160	700	8.6
6.1	160	800	7.7
6.1	160	900	6.7
6.1	160	1000	6

Conclusion

The results of this experiment show that as the frequency of the signal generator is increased, the current decreases inversely proportional to frequency. Therefore, if a graph of 1/frequency is created, it forms a straight line which passes through the origin as shown.


Effect of Frequency on Current in a Purely Inductive Circuit
Current (mA)
70
60
50
40
30
20
10
0
0
200
400
600
800
1000
1200
Frequency (Hz)


Effect of Frequency on Current in a Purely Inductive Circuit (1/f)
Current (mA)
70
60
50
40
30
20
10
0
0
0.002
0.004
0.006
0.008
0.01
0.012
1/Fequency (Hz)

Experiment 3 – Frequency and Current in a Purely Resistive Circuit

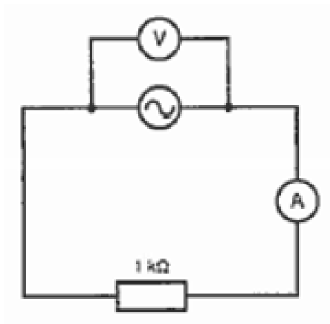
Aim

The aim of this experiment was to investigate what effect varying frequency had on the current in a purely resistive series circuit.

Underlying Physics

As I = V/R and neither the voltage or resistance are affected by frequency, the current remains constant.

Apparatus



- AC power supply
- Multimeter across power supply used to ensure the voltage across the power supply remains constant as well as accurately measure the frequency of the supply
- Ammeter to measure the current in the circuit
- 1 kΩ resistor

Method

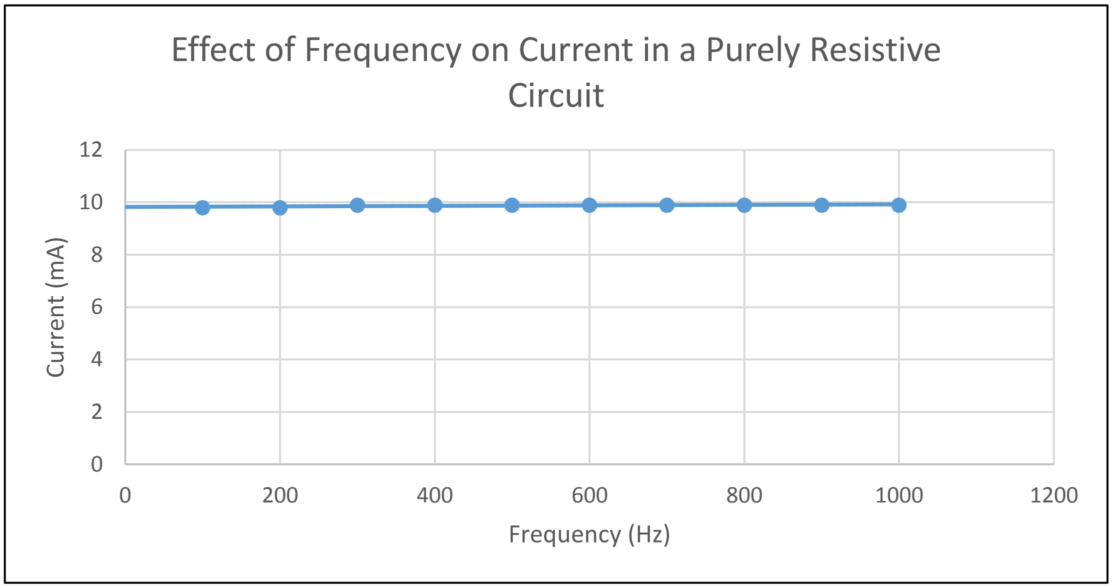
The method of this experiment was identical to the previous experiments, apart from the supply voltage being larger. The above circuit was set up as shown in the diagram and the frequency of the supply set to 100 Hz. The supply voltage was 9.9 V and the voltmeter was used to make sure it stayed constant at this voltage throughout the whole experiment. The current was measured on the ammeter and the experiment repeated a further nine times, increasing the frequency in increments of 100 Hz each time and recording the current with each different frequency.

Results

Voltage (V) ± 0.1V	Resistance (kΩ) ± 10%	Frequency (Hz) ± 1Hz	Current (mA) ± 0.1mA
9.9	1	100	9.8
9.9	1	200	9.8
9.9	1	300	9.9
9.9	1	400	9.9
9.9	1	500	9.9
9.9	1	600	9.9
9.9	1	700	9.9
9.9	1	800	9.9
9.9	1	900	9.9
9.9	1	1000	9.9

Conclusion

From the results if this experiment, it could be concluded that the frequency has no effect on the current in a purely resistive circuit, as shown in the graph below.



Experiment 4 – Variation of Frequency in a Series RLC Circuit
Aim

The aim of this experiment was to investigate what effect varying the frequency had on the current in a series RLC circuit.

Underlying Physics

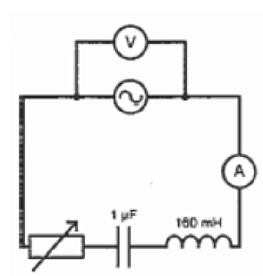
The equation for the impedance of a RLC circuit is given by:

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Hence, varying the frequency results in a change in the impedance of the circuit as both X_C and X_L are dependent on the frequency of the circuit as proven in experiments 1 and 2 respectively, whereas R can be assumed as independent of frequency, as proven in experiment 3.

As the frequency of the supply decreases, X_L will decrease and X_C will increase and so there is a certain frequency, where $X_C = X_L$, called the resonant frequency f_0. At this point, the impedance of the circuit is at its lowest point, being equal to the resistance of the circuit. Therefore, the circuit is acting purely as a resistive circuit, making the current in the circuit reach a maximum point. The current and voltage in the circuit are in phase at this point and there is said to be resonance.[4]

Apparatus



- AC power supply
- Multimeter across power supply used to ensure the voltage across the power supply remains constant as well as accurately measure the frequency of the supply
- Ammeter to measure the current in the circuit
- 1 µF capacitor
- 160 mH inductor

Method

For this experiment, the above circuit was assembled, with a supply voltage of 6.0 V. 4 different sets of results were taken, each with the circuit set up with different resistances. The first set of results were taken with the resistor set to 100 Ω. As with the previous experiments, the frequency of the supply started at 100 Hz and the reading on the ammeter recorded. This was repeated 9 more times, with the frequency being increased each time, and the current reading at each frequency recorded.

Once completed, this whole procedure was repeated a further 3 times increasing the resistance to 470 Ω, followed by 1 kΩ and finally 2.2 kΩ.

Results

Voltage (V) ± 0.1V	Resistance (Ω) ± 10%	Capacitance (μF) ± 10%	Inductance (mH) ± 10%	Frequency (Hz) ± 1Hz	Current (mA) ± 0.1mA
6.0	100	1	160	100	4.9
6.0	100	1	160	150	8
6.0	100	1	160	200	11.9
6.0	100	1	160	250	15.2
6.0	100	1	160	300	25
6.0	100	1	160	350	34.5
6.0	100	1	160	400	40.4
6.0	100	1	160	425	39.6
6.0	100	1	160	450	35.9
6.0	100	1	160	500	29.6

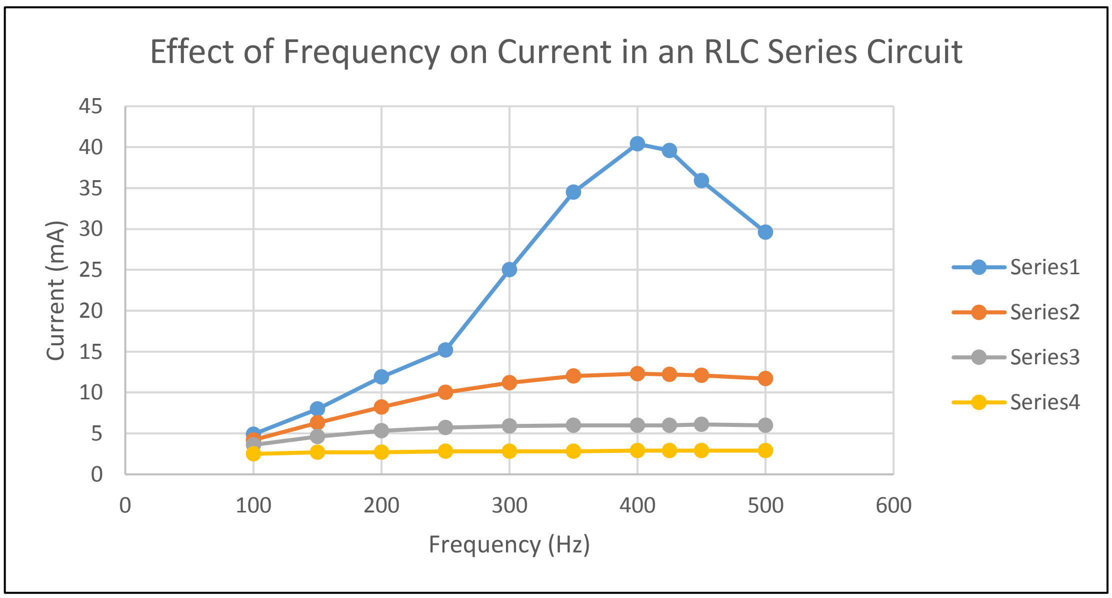
Voltage (V) ± 0.1V	Resistance (kΩ) ± 10%	Capacitance (μF) ± 10%	Inductance (mH) ± 10%	Frequency (Hz) ± 1Hz	Current (mA) ± 0.1mA
6.0	470	1	160	100	4.2
6.0	470	1	160	150	6.3
6.0	470	1	160	200	8.2
6.0	470	1	160	250	10
6.0	470	1	160	300	11.2
6.0	470	1	160	350	12
6.0	470	1	160	400	12.3
6.0	470	1	160	425	12.2
6.0	470	1	160	450	12.1
6.0	470	1	160	500	11.7

Voltage (V) ± 0.1V	Resistance (kΩ) ± 10%	Capacitance (μF) ± 10%	Inductance (mH) ± 10%	Frequency (Hz) ± 1Hz	Current (mA) ± 0.1mA
6.0	1	1	160	100	3.6
6.0	1	1	160	150	4.6
6.0	1	1	160	200	5.3
6.0	1	1	160	250	5.7
6.0	1	1	160	300	5.9
6.0	1	1	160	350	6
6.0	1	1	160	400	6
6.0	1	1	160	425	6
6.0	1	1	160	450	6.1
6.0	1	1	160	500	6

Voltage (V) ± 0.1V	Resistance (kΩ) ± 10%	Capacitance (μF) ± 10%	Inductance (mH) ± 10%	Frequency (Hz) ± 1Hz	Current (mA) ± 0.1mA
6.0	2.2	1	160	100	2.5
6.0	2.2	1	160	150	2.7
6.0	2.2	1	160	200	2.7
6.0	2.2	1	160	250	2.8
6.0	2.2	1	160	300	2.8
6.0	2.2	1	160	350	2.8
6.0	2.2	1	160	400	2.9
6.0	2.2	1	160	425	2.9
6.0	2.2	1	160	450	2.9
6.0	2.2	1	160	500	2.9

Conclusion

The results of this experiment show that as the frequency is increased, the current increases until a certain point, and the begins to decrease. This point is the resonant frequency which was found to 400 Hz ± 0.5% from this experiment.



Experiment 5 – Variation of Frequency in a Parallel RLC Circuit
Aim

The aim of this experiment was to investigate what effect varying frequency had on the current in a parallel RLC circuit.

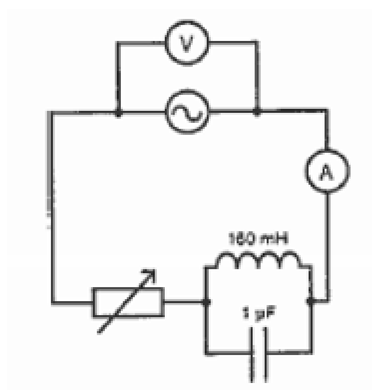
Underlying Physics

Resonance frequency also occurs in parallel RLC circuits but in this case the current reaches a minimum value at this frequency. At this frequency, the current and voltage are completely out of phase and so , according to the relationship

$$Z = \sqrt{R^2 + \left(\frac{1}{(X_L + X_C)^2}\right)}$$

Since X_C and X_L will have opposite signs (one will be positive and one negative), the reactance and the impedance of the circuit will be at a maximum, which will cause the current to be a minimum at resonance.

Apparatus



- AC power supply
- Multimeter across power supply used to ensure the voltage across the power supply remains constant as well as accurately measure the frequency of the supply
- Ammeter to measure the current in the circuit
- Variable resistor
- 1 μF capacitor
- 160 mH inductor

Method

For this experiment, the above circuit was assembled, with a supply voltage of 6.0 V. 4 different sets of results were taken, each with the circuit set up with different resistances. The first set of results were taken with the resistor set to 100 Ω. As with the previous experiments, the frequency of the supply started at 100 Hz and the reading on the ammeter recorded. This was repeated 9 more times, with the frequency being increased each time, and the current reading at each frequency recorded.

Once completed, this whole procedure was repeated a further 3 times increasing the resistance to 470 Ω, followed by 1 kΩ and finally 2.2 kΩ.

Results

Voltage (V) ± 0.1V	Resistance (Ω) ± 10%	Capacitance (μF) ± 10%	Inductance (mH) ± 10%	Frequency (Hz) ± 1Hz	Current (mA) ± 0.1mA
6.0	100	1	160	100	31.7
6.0	100	1	160	150	26.2
6.0	100	1	160	200	19.5
6.0	100	1	160	250	14.5
6.0	100	1	160	300	9.7
6.0	100	1	160	350	5.3
6.0	100	1	160	400	3.4
6.0	100	1	160	425	4.4
6.0	100	1	160	450	7.1
6.0	100	1	160	500	8.4

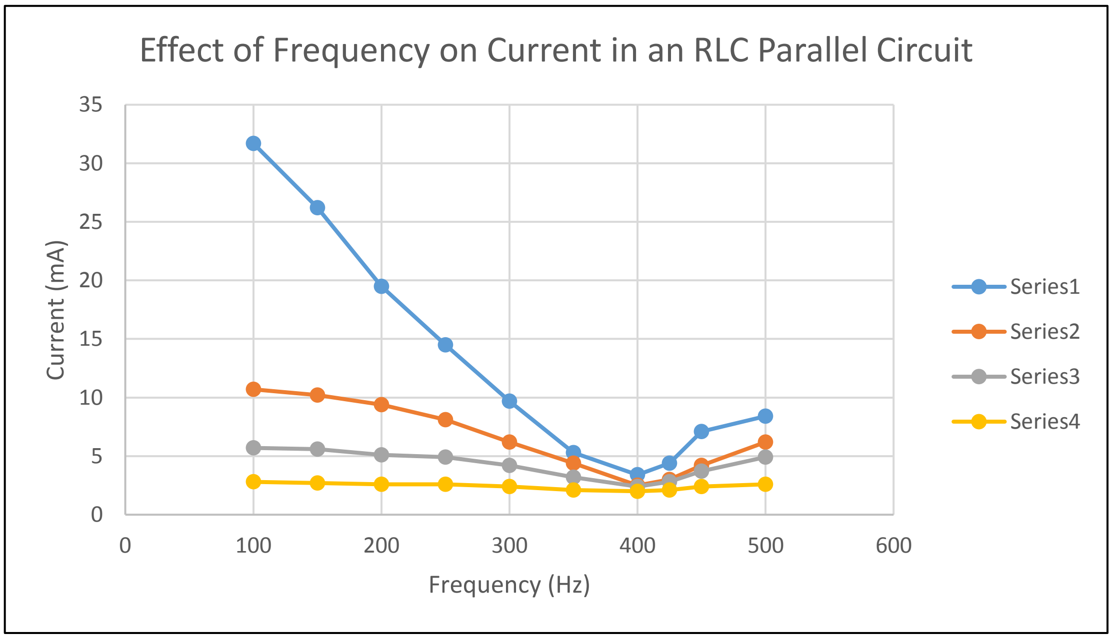
Voltage (V) ± 0.1V	Resistance (kΩ) ± 10%	Capacitance (μF) ± 10%	Inductance (mH) ± 10%	Frequency (Hz) ± 1Hz	Current (mA) ± 0.1mA
6.0	470	1	160	100	10.7
6.0	470	1	160	150	10.2
6.0	470	1	160	200	9.4
6.0	470	1	160	250	8.1
6.0	470	1	160	300	6.2
6.0	470	1	160	350	4.4
6.0	470	1	160	400	2.5
6.0	470	1	160	425	3
6.0	470	1	160	450	4.2
6.0	470	1	160	500	6.2

Voltage (V) ± 0.1V	Resistance (kΩ) ± 10%	Capacitance (μF) ± 10%	Inductance (mH) ± 10%	Frequency (Hz) ± 1Hz	Current (mA) ± 0.1mA
6.0	1	1	160	100	5.7
6.0	1	1	160	150	5.6
6.0	1	1	160	200	5.1
6.0	1	1	160	250	4.9
6.0	1	1	160	300	4.2
6.0	1	1	160	350	3.2
6.0	1	1	160	400	2.4
6.0	1	1	160	425	2.8
6.0	1	1	160	450	3.7
6.0	1	1	160	500	4.9

Voltage (V) ± 0.1V	Resistance (kΩ) ± 10%	Capacitance (μF) ± 10%	Inductance (mH) ± 10%	Frequency (Hz) ± 1Hz	Current (mA) ± 0.1mA
6.0	2.2	1	160	100	2.8
6.0	2.2	1	160	150	2.7
6.0	2.2	1	160	200	2.6
6.0	2.2	1	160	250	2.6
6.0	2.2	1	160	300	2.4
6.0	2.2	1	160	350	2.1
6.0	2.2	1	160	400	2
6.0	2.2	1	160	425	2.1
6.0	2.2	1	160	450	2.4
6.0	2.2	1	160	500	2.6

Conclusion

The results of this experiment show that as the frequency is increased, the current decreases until a certain point, and the begins to increase. This point is the resonant frequency which was found to 400 Hz ± 0.5% as with the previous experiment.



Theoretical resonant frequency

To confirm the resonant frequency observed in the previous two experiments, X_C and X_L were equated to give the equation:

$$2\pi fL = \frac{1}{2\pi fC}$$
$$f^2 = \frac{1}{4\pi^2 LC}$$
$$f = \frac{1}{2\pi\sqrt{LC}}$$ [5]

Substituting in the values for their respected letters gave:

$$f = \frac{1}{2\pi\sqrt{0.16\times1\times10^{-6}}}$$
$$f = 397.9Hz$$

Conclusion

From my experimental data, I have come to the following conclusions:

- In a purely resistive circuit, frequency has no effect on current
- In a purely capacitive circuit, the current is directly proportional to the frequency as current increases as frequency increase.
- In a purely inductive circuit, the frequency is inversely proportional to current as current decreases as frequency increases
- In a series RLC circuit, the current increases until the resonance frequency is reached, and then begins to fall. I found the resonant frequency of this circuit to be 400 ± 0.5% Hz
- In a parallel RLC circuit, the current decreases until the resonant frequency is reached, and then begins to rise. I found the resonant frequency in this circuit to be the same as in the series RLC circuit.
- The theoretical resonant frequency was calculated to be 397.9 ± 7% Hz.

References

1. Duncan T, Advanced Physics: Fields, waves and Atoms, London, John Murray, 1979 pp165
2. Duncan T, Advanced Physics: Fields, waves and Atoms, London, John Murray, 1979 pp167
3. Duncan T, Advanced Physics: Fields, waves and Atoms, London, John Murray, 1979 pp173
4. Duncan T, Advanced Physics: Fields, waves and Atoms, London, John Murray, 1979 pp174
5. Duncan T, Advanced Physics: Fields, waves and Atoms, London, John Murray, 1979 pp175