Advanced Higher Physics

This investigation was created to accurately measure the refractive index of a medium through different methods and evaluate these experiments accordingly.

Measuring the Refractive Index

Comparing Methods
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</table>
Abstract

This investigation will compare methods of measuring the refractive index of a medium and evaluate these experiments accordingly. All three methods measured the refractive index with some degree of accuracy and precision.

Method 1 using a concave mirror was the most simple to undertake and produced accurate results (most accurate being 1.32±0.04) for which the given value was within the maximum uncertainty.

Method 2 using a travelling microscope was more difficult to carry out but allowed the refractive index of both a liquid and a solid, i.e. glass or Perspex. This method resulted in values lower than the accepted value (1.43±0.12) for glass which shows a systematic error but was still accurate enough for the accepted value to lie within the uncertainties of the experiment. Measuring the refractive index of water was a lot more inconsistent than the glass.

Method 3 also can only measure the refractive index of a small volume of liquid, not solid. Although it produced the most accurate results but required custom building of apparatus. The uncertainties were quite large though (1.32±0.09) which shows the experiment could have a lot of errors.

Underlying Physics

Light travels at different speeds through different mediums. The speed that light travels at is dependent on the refractive index of the medium. Take a red laser shining at an angle into a glass block as shown below.
Air has a refractive index of 1.0003, rounded to 1, and glass has a considerably higher refractive index of around 1.5 depending on the type of glass. This causes the light ray to slow as it enters the medium and increase in speed as it leaves the medium. Snell’s Law states that the refractive index of the first medium, \( n_1 \), divided by the refractive index of the second medium, \( n_2 \), is equal to \( \sin \theta_1 \) divided by \( \sin \theta_2 \), or:

\[
\frac{n_2}{n_1} = \frac{\sin \theta_1}{\sin \theta_2}
\]

As previously stated, the refractive index of air is 1 which allows for the equation to be simplified to,

\[
n_2 = \frac{\sin \theta_1}{\sin \theta_2}
\]

When water is poured into the mirror (method 1) the light will be refracted and the reflected ray will return to a different position. Finding the point of non-parallax allows measurements to be taken that can calculate refractive index using Snell’s Law.
Parallax involves the use of two different points of view. This could be a human’s eyes or cameras on either side of the world. When an object is seen from two points of view, it moves relative to its background and surrounding objects. Measuring the angle at which these lines of sight view the object can determine how far away the object is. This is how humans and most predatory animals perceive depth and astronomers measure the distance to closer stars. When looking at the pin and the reflected image in a concave mirror, moving the view position will cause the pin and the reflected image to move different distances. The point of non-parallax occurs when both the movement of the reflected image of the pin and the real pin coincide. The distance between the image and the pin should never change, as shown in the diagram below, with the black pin being the start position of the pin. The red pin occurs when the view position is moved left to right. Using the point of non-parallax in this experiment allows for the exact change in distance that the reflected ray takes due to the water in the mirror.
Determination of the refractive index of water using a concave mirror (Method 1)

The following experiment was carried out to investigate refractive using a concave mirror (Tyler, 1959, pp. 76-77).

Underlying Physics

Snell’s Law states,

\[
\frac{\sin i}{\sin r} = n_i
\]

Which can be said to equal,

\[
\frac{\sin \angle QC A}{\sin \angle QCA} = n_i
\]

\[
\frac{QA}{QC} = \frac{QA}{QC} = n_i
\]

Hence,

\[
n_i = \frac{QC}{QC}
\]

Since the rays entering the eye are close to the axis and are a small pencil of rays the formula can be written as,

\[
n_i = \frac{AC}{AC}
\]
Apparatus

- Concave mirror
- Liquid (water)
- Retort stand
- Clamp
- Pin
- Metre rule

Procedures

A pin is clamped above a concave mirror so that the reflexion coincides with the real pin at point C. The real pin and the projected image must be at the point of no parallax. The height CP is now measured. The concave mirror is then filled with water, enough to overcome the surface tension of the water and the curvature of the mirror. The pin is again adjusted so the new reflexion and the pin are again at the point of no parallax. The new height C'P is now measured. The experiment is repeated using mirrors with different focal lengths.

Data

<table>
<thead>
<tr>
<th>Focal Length of concave mirror (cm)</th>
<th>Distance CA (cm)</th>
<th>Distance C'A (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>23.1</td>
<td>18.1</td>
</tr>
<tr>
<td>15</td>
<td>30.1</td>
<td>23.4</td>
</tr>
<tr>
<td>20</td>
<td>38.6</td>
<td>29.3</td>
</tr>
</tbody>
</table>
### Analysis:

Mirror with focal length of 10 cm,  

\[ CA = 23.1 \]

\[ C'A = 18.1 \]

\[ \therefore n_i = \frac{CA}{C'A} = \frac{23.1}{18.1} = 1.28 \]

<table>
<thead>
<tr>
<th>Focal Length of concave mirror (cm)</th>
<th>Calculated refractive Index of Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1.28±0.06</td>
</tr>
<tr>
<td>15</td>
<td>1.29±0.05</td>
</tr>
<tr>
<td>20</td>
<td>1.32±0.04</td>
</tr>
<tr>
<td>30</td>
<td>1.34±0.03</td>
</tr>
<tr>
<td>Inconsistent Focal Length (Flexible Mirror)</td>
<td>1.25±0.02</td>
</tr>
</tbody>
</table>
Inconsistent Focal Length (Flexible Mirror)
Uncertainties

Uncertainty is distance $CA$ for data using mirror of focal length 10 cm:

Metre rule calibration uncertainty = ±0.5cm

Reading uncertainty = ±0.5cm

Total uncertainty:

\[ \sqrt{(0.5)^2 + (0.5)^2} = \sqrt{0.5} = \pm 0.7 \text{ cm} \]

Total percentage uncertainty:

\[ \frac{0.7}{23.1} \times 100 = 3.03\% \]

Uncertainty in distance $C'A$ for data using mirror of focal length 10cm:

Metre rule calibration uncertainty = ±0.5cm

Reading uncertainty = ±0.5cm

Total uncertainty:

\[ \sqrt{(0.5)^2 + (0.5)^2} = \sqrt{0.5} = \pm 0.7 \text{ cm} \]

Total percentage uncertainty:

\[ \frac{0.7}{18.1} \times 100 = 3.87\% \]

Uncertainty in refractive index:

\[ \sqrt{(3.03)^2 + (3.87)^2} \]
\[
= \sqrt{24.16}
\]
\[
= \pm 4.9\%
\]

Therefore total absolute uncertainty for the refractive index:

\[
= \frac{1.28}{100} \times 4.9
\]
\[
= \pm 0.06
\]

**Conclusion:**

As the given value for the refractive index of water is 1.33, the most accurate mirrors also had the longest focal length. The mirror with the longest focal length, 30 cm, gave the refractive index to be 1.34±0.03, extremely close to the given value. The mirror with a focal length of 20 cm, gave the refractive index of 1.32±0.04, equally close to the given value. The least accurate of all the mirrors was the flexible mirror, with a result of 1.25±0.02.

**Evaluation of Procedures:**

Using a ruler to measure the distance from the mirror to the pin can be inaccurate because the flat edge of the ruler will not sit flush to the curved mirror, leading to a small distance being measured. This could be reduced if a thin metal rod was used and marked to the height of the pin, and then measured with a ruler.

A flexible mirror is definitely inaccurate as the concavity changes throughout the mirror due to years in storage. This causes the mirror to have no point of non-parallax and in turn causes the experiment to fail. As the observer moved to find the point of non-parallax the concavity changed, making the point of non-parallax change. The point closest to the point of non-parallax was used instead and produced inaccurate results as expected.

The experiment produced a refractive index close to that of water, excluding the flexible mirror. The refractive indices measured with mirrors that had longer focal lengths were more accurate than mirrors with short focal lengths. This could be because a misjudgement when finding the point of non-parallax which is found by eye thus may be inaccurate and will be more prominent at smaller focal lengths.
Determination of the Refractive Index of Glass and a Liquid using a Travelling Microscope (Method 2)

The following experiment was carried out to investigate refractive index using a travelling microscope.

Underlying Physics

The travelling microscope allows for the measurement of very small distances. The difference in distance travelled by a ray of light passing through air and a ray of light passing thorough glass can be measured. Again Snell’s law can be used to determine the refractive index.

Snell’s law states:

\[ \frac{\sin i}{\sin r} = n_a \]

Which can be said to equal,

\[ \frac{\sin \angle APQ}{\sin \angle AP'Q} = n_a \]

\[ \frac{AQ}{PQ} = n_a \]

\[ \frac{AQ}{P'Q} = n_a \]
Hence,

\[ n_g = \frac{1}{n_a} = \frac{PQ}{P'Q} \]

For extremely small angles PQ and P’Q can be equal PA and P’A respectively, as the distance AQ is negligible.

\[ \therefore n_g = \frac{PA}{P'A} = \frac{\text{real thickness}}{\text{apparent thickness}} \]

Apparatus:
- Travelling microscope with Vernier scale
- Glass block
- Tank with glass sides
- Lycopodium powder
- Fine sand

Description of Procedures:

A piece of paper is placed on the bench with and marked with an ink spot ‘P’. A travelling microscope is adjusted to bring P into focus and arranged to move vertically. Arranging the microscope to move vertically in one direction prevents ‘flyback’ which can cause an error with the measurement. An initial reading on the Vernier scale is taken. The glass block is now placed in a position over P, between the paper and the travelling microscope. The microscope is now positioned and re-adjusted to bring P into focus. The Vernier is read at P’. An ink spot is made on the top of the glass block and the travelling microscope is adjusted to bring it into focus. Finally the Vernier is read (A).

To measure the refractive index of a liquid, the same procedure is taken except the first and second reading is taken on sand particles at the bottom of the tank (the first has no liquid in the tank) and the third is taken on the particles of lycopodium powder floating on the surface of the liquid.

Data:

<table>
<thead>
<tr>
<th>Measuring the Refractive Index of Water</th>
<th>Try 1</th>
<th>Try 2</th>
<th>Try 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement No.1 (mm) (1)</td>
<td>99.0</td>
<td>99.0</td>
<td>100.0</td>
</tr>
<tr>
<td>Measurement No.2 (mm) (2)</td>
<td>100.6</td>
<td>100.1</td>
<td>100.9</td>
</tr>
<tr>
<td>Measurement No.3 (mm) (3)</td>
<td>104.5</td>
<td>105.6</td>
<td>106.3</td>
</tr>
</tbody>
</table>
Measuring the Refractive Index of Glass

<table>
<thead>
<tr>
<th></th>
<th>Try 1</th>
<th>Try 2</th>
<th>Try 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement No.1 (mm) (1)</td>
<td>98.7</td>
<td>101.0</td>
<td>102.3</td>
</tr>
<tr>
<td>Measurement No.2 (mm) (2)</td>
<td>99.0</td>
<td>101.3</td>
<td>102.6</td>
</tr>
<tr>
<td>Measurement No.3 (mm) (3)</td>
<td>99.7</td>
<td>102.0</td>
<td>103.3</td>
</tr>
</tbody>
</table>

Analysis:

As

\[ n_g = \frac{PA}{P^*A} = \frac{\text{real thickness}}{\text{apparent thickness}} \]

For Try 1, measuring the refractive index of water,

\[ n_i = \frac{(3) - (1)}{(3) - (2)} = \frac{104.5 - 99.0}{104.5 - 100.6} = \frac{5.5}{3.9} = 1.41 \]

For Try 1, measuring the refractive index of glass,

\[ n_i = \frac{(3) - (1)}{(3) - (2)} = \frac{99.7 - 98.7}{99.7 - 99.0} = \frac{1}{0.7} = 1.43 \]

Measuring the Refractive Index of Water

<table>
<thead>
<tr>
<th></th>
<th>Try 1</th>
<th>Try 2</th>
<th>Try 3</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement No.1 (mm) (1)</td>
<td>1.41±0.21</td>
<td>1.20±0.13</td>
<td>1.17±0.13</td>
<td>1.26</td>
</tr>
</tbody>
</table>

Measuring the Refractive Index of Glass

<table>
<thead>
<tr>
<th></th>
<th>Try 1</th>
<th>Try 2</th>
<th>Try 3</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement No.1 (mm) (1)</td>
<td>1.43±0.12</td>
<td>1.43±0.12</td>
<td>1.43±0.12</td>
<td>1.43</td>
</tr>
</tbody>
</table>
Determination of the Refractive Index of Glass and a Liquid using a Travelling Microscope (Method 2)
Uncertainties:

Reading uncertainty for travelling microscope,

\[ \frac{1}{2} \text{ of the smallest division} \]
\[ = \frac{1}{2} \times 0.1 mm \]
\[ = 0.05 mm \]

Random uncertainties:

random uncertainty = \( \frac{\text{max measurement} - \text{min measurement}}{n} \)

For distance (3) – (1) in refractive index of water,

\[ \frac{6.6 - 5.5}{3} \]
\[ = \pm 0.37 mm \]

For distance (3) – (2) in refractive index of water,

\[ \frac{5.5 - 3.9}{3} \]
\[ = \pm 0.53 mm \]

For distance (3) – (1) in refractive index of glass,

\[ \frac{1 - 1}{3} \]
\[ = \pm 0 mm \]

For distance (3) – (2) in refractive index of glass,

\[ \frac{0.7 - 0.7}{3} \]
\[ = \pm 0 mm \]
Combined uncertainties:

For distance (3)-(1) in refractive index of water,

\[ = \sqrt{(0.37)^2} \]

As the reading uncertainty is less than \( \frac{1}{3} \) of the random uncertainty it can be ignored,

\[ = \pm 0.37 \text{ mm} \]

For distance (3)-(2) in refractive index of water,

\[ = \sqrt{(0.53)^2} \]

As the reading uncertainty is less than \( \frac{1}{3} \) of the random uncertainty it can be ignored,

\[ = \pm 0.53 \text{ mm} \]

For distance (3)-(1) in refractive index of glass,

\[ = \sqrt{(0.05)^2} \]

As the reading uncertainty is less than \( \frac{1}{3} \) of the random uncertainty it can be ignored,

\[ = \pm 0.05 \text{ mm} \]

For distance (3)-(2) in refractive index of glass,

\[ = \sqrt{(0.05)^2} \]

As the reading uncertainty is less than \( \frac{1}{3} \) of the random uncertainty it can be ignored,

\[ = \pm 0.05 \text{ mm} \]

Total uncertainties:

For refractive index of water Try 1,

\[ \% \text{ uncertainty} = \sqrt{(\% \text{ uncertainty in (3) - (1)})^2 + (\% \text{ uncertainty in (3) - (2)})^2} \]
\[
= \sqrt{(6.73)^2 + (13.59)^2}
= \sqrt{229.97}
= 15.16\%
\]

absolute uncertainty = \( \frac{1.41}{100} \times 15.16 \)

= \pm 0.21

Conclusion:

The results for the refractive index of water were not very precise as shown in the graph but did average out at 1.26 which is only 0.07 away from the given value. The results for the refractive index of glass were extremely precise (all three results being 1.43\pm0.12) but still fairly inaccurate, again averaging to be 0.07 away from the given value of the refractive index of glass of 1.5.

Evaluation of procedures:

The travelling microscope used could not slide as far as the width of the glass block in the apparatus diagram. A glass microscope slide was used instead which is much thinner which allowed the travelling microscope to easily take measurements. It was also extremely uniform and gave very precise results.

The glass tank pictured has a curved bottom which meant the sand particles on the bottom of the tank were different distances from the surface and the lycopodium powder granules. To avoid this, the microscope was kept still when adding the lycopodium powder to make sure the measurements were correct. To avoid this complication a glass tank with a flat bottom could have been used but due to limited resources this was not possible.
Determination of the refractive Index of a small volume of water using Wollaston’s Method (Method 3)

The following experiment was carried out to investigate refractive using Wollaston’s method (Tyler, 1959, pp. 74-75)

Underlying Physics

To find $\theta$, we simply use trigonometry,

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\theta = \tan^{-1} \left( \frac{h_1 - h_2}{x} \right)$$

By Snell’s law we have,

$$\sin \theta = n_s \sin A$$

Since $A = 90 - B$,

$$\sin \theta = n_s \cos B$$
Also,

\[ \frac{\sin C}{\sin 90} = \frac{n_t}{n_s} = n_t \]

\[ n_t = n_s \sin C \]

From both these equation we can square and add to get,

\[ n_t = \sqrt{n_s^2 - \sin^2 \theta} \]

Thus the refractive index of the liquid can be found if the refractive index of the glass block is known.

**Apparatus:**

- Sodium lamp
- Ground glass sheet
- Liquid (water)
• Glass block with horizontal scratch on one face
• Blackened baseboard with upright attached at 90° and near the upper end of the upright a slit for observation

Description of Procedures:

The apparatus was set up as shown above. A small amount of water was poured onto the base board, and the glass cube is placed on the water with the scratch facing towards the observation slit. When looking through the narrow slit at the cube while it is some distance away, the viewer should see the surface of the glass that is touching the water illuminated brightly. Upon moving the cube closer to the observation slit the illuminated surface will become darker. The cube is moved into a position where the line of demarcation between the dark and the light fields coincides with the scratch on the glass block. The distance of the observation slit and the scratch from the baseboard are measured ($h_1$ and $h_2$ respectively). Also the distance from the upright board to the nearest face of the cube is measured ($x$).

Data:

<table>
<thead>
<tr>
<th></th>
<th>Try 1</th>
<th>Try 2</th>
<th>Try 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_1$ (cm)</td>
<td>24.5</td>
<td>24.5</td>
<td>24.5</td>
</tr>
<tr>
<td>$h_2$ (cm)</td>
<td>1.1</td>
<td>1.1</td>
<td>1.1</td>
</tr>
<tr>
<td>$x$ (cm)</td>
<td>22.7</td>
<td>22.3</td>
<td>21.6</td>
</tr>
</tbody>
</table>

Analysis:

\[
\theta = \tan^{-1} \frac{h_1 - h_2}{x} \\
\theta = \tan^{-1} \frac{24.5 - 1.1}{22.7} \\
\theta = 45.87°
\]
Using the refractive index of the glass cube as 1.5,

\[ n_t = \sqrt{n_e^2 - \sin^2 \theta} \]

\[ n_t = \sqrt{(1.5)^2 - \sin^2 (45.87)} \]

\[ n_t = \sqrt{1.735} \]

\[ n_t = 1.32 \]

<table>
<thead>
<tr>
<th></th>
<th>Try 1</th>
<th>Try 2</th>
<th>Try 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculated Refractive Index</td>
<td>1.32±0.09</td>
<td>1.31±0.09</td>
<td>1.31±0.09</td>
</tr>
</tbody>
</table>

Determination of the Refractive Index of a small volume of water using Wollaston's Method (Method3)
Uncertainties:

Metre rule calibration uncertainty = ±0.5mm

Reading uncertainty = ±0.5mm

Combined uncertainties:

\[
= \sqrt{(0.5)^2 + (0.5)^2} = \sqrt{0.5} = ±0.71 \text{mm}
\]

% uncertainty = \(\frac{0.071}{22.7}\) × 100 = 0.31%

% uncertainty = \(\frac{0.071}{24.5}\) × 100 = 0.29%

% uncertainty = \(\frac{0.071}{1.1}\) × 100 = 6.45%

% total uncertainty = \(\sqrt{(6.45)^2 + (0.29)^2 + (0.31)^2}\) = \(\sqrt{41.8}\) = ±6.46%

\[
= \frac{1.32}{100} \times 6.46 = ±0.09
\]

Evaluation of procedures:

Custom building of the baseboard and the upright took time and was laborious as it is not a standard piece of equipment. The sodium lamp also took some time to warm up before the experiment could begin which should be taken into account when conducting the experiment. To view the reflexion of the sodium lamp the laboratory had to be dimmed or blacked out which is not ideal if sharing with other students.
Overall Conclusion

All three methods have strengths and weaknesses. The most accurate experiment was method 3, although it was not the simplest to set up. The simplest had to be method 1 which required few pieces of apparatus and was easy to carry out. However method 2 was the only experiment to measure both the refractive index of a liquid and the refractive index of a solid. Overall method 3 proved to be the best as accuracy is key when carrying out experiments and once the baseboard was built the experiment was fairly straightforward to carry out.

Overall Evaluation

All three experiments worked well to give accurate results. To make the results even more reliable, each experiment could be carried out again and the results averaged. For method 1, a concave mirror with a focal length of 25cm could have been bought and used instead of the flexible mirror to give a more standard set of results but due to resources and time this was not an option. Also another experiment could be used, such as ‘Determination of the index of refraction using a laser pointer’ (source: http://www.cuhou.net/index.php/exercises-mainmenu-13/classroom-experiments-and-activities-mainmenu-186/203-determination-of-the-index-of-refraction-using-a-laser-pointer) to compare results with other experiments, adding to the degree of validity.
References:

1. http://hyperphysics.phy-astr.gsu.edu/hbase/geopt/refr.html#c3  22 Mar. 16