Question 1(b)

Candidate 1 evidence

b. $f(x) = \dfrac{1 - x^2}{1 + 4x^2}$

$$f'(x) = \frac{2x\,(1+4x^2) - (1-x^2)\cdot 8x}{(1+4x^2)^2}$$

$$= \frac{2x + 8x^3 - 8x + 8x^3}{(1+4x^2)^2}$$

$$= \frac{16x^3 - 6x}{(1+4x^2)^2}$$

Candidate 2 evidence

1b) $f(x) = \dfrac{1-x^2}{1+4x^2}$ $\begin{matrix} u \\ v \end{matrix}$ $\quad u' = -2x$, $v' = 8x$

$$f'(x) = \frac{(1+4x).-2x - 8x(1-x^2)}{(1+4x^2)^2}$$

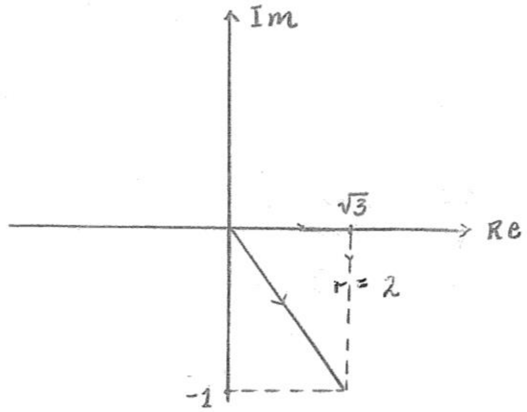
$$f'(x) = \frac{-2x - 8x^2 - 8x + 8x^3}{(1+4x^2)^2}$$

$$f'(x) = \frac{8x^3 - 8x^2 - 10x}{(1+4x^2)^2} = \frac{2x(4x^2 - 4x - 5)}{(1+4x^2)^2}$$

Question 8

Candidate 3 evidence

8\. a)



b) $r = \sqrt{(\sqrt{3})^2 + (-1)^2} = 2$

$\theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

$z = 2\left(\cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)\right)$

$= 2\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$

$w = az = 2a\left(\cos\frac{\pi}{6} - i\sin\frac{\pi}{6}\right)$

$[r(\cos\theta + i\sin\theta)]^n = r^n(\cos n\theta + i\sin n\theta)$

c) $w^8 = (2a)^8\left(\cos 8\cdot\frac{\pi}{6} - i\sin 8\cdot\frac{\pi}{6}\right)$

$= 256a^8\left(\cos\frac{4\pi}{3} - i\sin\frac{4\pi}{3}\right)$

$= 256a^8\left(\cos -\frac{2}{3}\pi - i\sin -\frac{2}{3}\pi\right)$

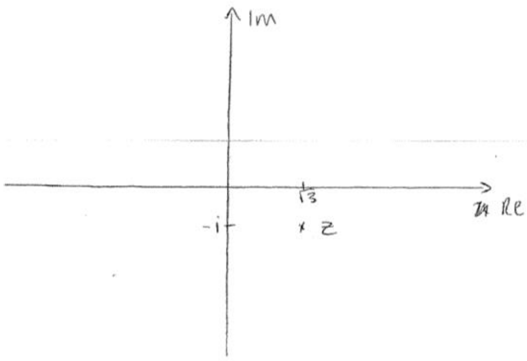
$= 256a^8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

$= 256a^8\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

Candidate 4 evidence

ENTER NUMBER OF QUESTION

8a.



$z = (\sqrt{3}, -i)$

b.

$w = az$

$= a(\sqrt{3} - i)$

$=$

$r^n(\cos n\theta + i\sin n\theta)$

$r = \sqrt{\sqrt{3}^2 + (-1)^2}$

$= 2$

$\theta = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

$= -\frac{\pi}{6}$

$= \frac{7\pi}{6}$

S A

T C

~~8c) $w^8 = (az)^8$~~

$w = a\left[2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)\right]$

$= a\left(\cos\frac{7\pi}{3} + i\sin\frac{7\pi}{3}\right)$

c.

$w^8 = (az)^8$

$= \left(a\left(\cos\frac{7\pi}{3} + i\sin\frac{7\pi}{3}\right)\right)^8$

$= a^8\left(\cos 8\left(\frac{7\pi}{3}\right) + i\sin 8\left(\frac{7\pi}{3}\right)\right)$

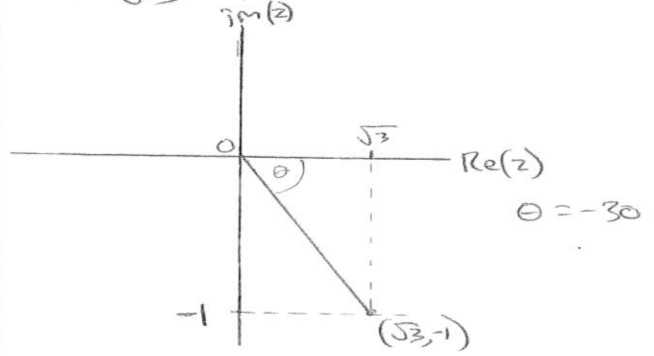
$= a^8\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)$

$= 2a^8(-1 + i\sqrt{3})$

Candidate 5

ENTER NUMBER OF QUESTION

8 a) $z = \sqrt{3} - i$



$\theta = -30$

b) $z = \sqrt{3} - i$

$az = \omega = a\sqrt{3} - ai$

$r = |z| = \sqrt{(a\sqrt{3})^2 + (-a)^2}$

$r = \sqrt{3a^2 + a^2}$

$r = \sqrt{4a^2}$

$r = 2a$

$\arg(z) = \tan^{-1}\left(\frac{-1a}{\sqrt{3}a}\right)$

$\arg(z) = -30$

$\underline{\omega = 2a(\cos(-30) + i\sin(-30))}$

c) $\omega^8 = (2a)^8(\cos(-30 \times 8) + i\sin(-30\times 8))$

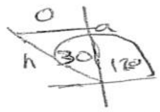
$\omega^8 = 256a^8(\cos(-240) + i\sin(-240))$

$\omega^8 = 256a^8(\cos(120) + i\sin(120))$

$\omega^8 = 128a^8 + i\,128\sqrt{3}\,a^8$

$\omega^8 = 128a^8(1 + \sqrt{3}i)$

$\underline{\omega^8 = 128a^8(1 + i\sqrt{3})}$



$\sin 30 = \frac{o}{h}$

$h \times \sin 30 = o$

$\text{real}(z) = 256a^8 \times 1/2$

$\text{real}(z) = 128a^8$

$\text{im}(z) = 256a^8 \times \cos 30$

$\text{im}(z) = \frac{256\sqrt{3}}{2} \times a^8$

Question 3

Candidate 6

3

$$\left(\frac{3}{x} - 2x\right)^{13}$$

$$= \binom{13}{0}\left(\frac{3}{x}\right)^{13}(-2x)^0 + \binom{13}{1}\left(\frac{3}{x}\right)^{12}(-2x)^1 + \binom{13}{2}\left(\frac{3}{x}\right)^{11}(-2x)^2 +$$

$$\binom{13}{3}\left(\frac{3}{x}\right)^{10}(-2x)^3 + \binom{13}{4}\left(\frac{3}{x}\right)^{9}(-2x)^4 + \binom{13}{5}\left(\frac{3}{x}\right)^{8}(-2x)^5 +$$

$$\binom{13}{6}\left(\frac{3}{x}\right)^{7}(-2x)^6 + \binom{13}{7}\left(\frac{3}{x}\right)^{6}(-2x)^7 + \binom{13}{8}\left(\frac{3}{x}\right)^{5}(-2x)^8 +$$

$$\binom{13}{9}\left(\frac{3}{x}\right)^{4}(-2x)^9 + \binom{13}{10}\left(\frac{3}{x}\right)^{3}(-2x)^{10} + \binom{13}{11}\left(\frac{3}{x}\right)^{2}(-2x)^{11} +$$

$$\binom{13}{12}\left(\frac{3}{x}\right)^{1}(-2x)^{12} + \binom{13}{13}\left(\frac{3}{x}\right)^{0}(-2x)^{13}.$$

$$= 1 \times \frac{1594323}{x^{13}} - \frac{13817466}{x^{11}} + \frac{55269864}{x^{9}}$$

$$- \frac{135104112}{x^{7}} + \frac{225173520}{x^{5}} - \frac{270208224}{x^{3}} +$$

$$\frac{240185088}{x} - 160123392x + 80061696x^3$$

$$-29652480x^5 + 7907328x^7 - 1437696x^9$$

$$+ 159744x^{11} - 892x^{13}$$

term in $x^9 = \underline{-1437696}$

Candidate 7

ENTER NUMBER OF QUESTION

3) General term $= \binom{13}{r}\left(\frac{3}{x}\right)^{13-r}(-2x)^r$

$$= \binom{13}{r}\left(\frac{3}{x}\right)^{13}\left(\frac{3}{x}\right)^{-r}(-2x)^r$$

$$= \binom{13}{r}(3)^{13}\left(\frac{1}{x}\right)^{13}\left(\frac{3}{x}\right)^{-r}(-2x)^r$$

$$= \binom{13}{r}1594323(3)^{-r}(-2)^r\left(\frac{1}{x}\right)^{13}\left(\frac{1}{x}\right)^{-r}(x)^r$$

$$= \binom{13}{r}1594323(3)^{-r}(-2)^r\left(\frac{1}{x}\right)^{13-r}(x)^r$$

$$= \binom{13}{r}1594323\left(\frac{-2^r}{3^r}\right)\left(\frac{x^r}{x^{13-r}}\right)$$

when $r=11$: General term $= \binom{13}{11}1594323\left(\frac{-2^{11}}{3^{11}}\right)\left(\frac{x^{11}}{x^{13-11}}\right)$

$$= 78.1594323\left(\frac{-2^{11}}{3^{11}}\right)x^9$$

$$= -1437696x^9$$

Question 2

Candidate 8

2a) $U_n = ar^{n-1}$

$U_2 = 108$

$U_5 = 4$

$108 = ar^1$

$4 = ar^3$

$\frac{4}{r^3} = a$

$108 = \frac{4\cancel{r}}{r^{\cancel{3}2}}$

$a = \frac{108}{r}$

$4 = \frac{108}{\cancel{r}} \cdot r^{\cancel{3}2}$

$4 = 108r^2$

$r^2 = \frac{1}{27}$

$r = \frac{1}{\sqrt{27}}$

b) As $r < 1$ there is a sum to infinity.

$\frac{1}{\sqrt{27}} < 1$

$a = \frac{108}{\frac{1}{\sqrt{27}}}$

c) $S_\infty = \frac{a}{1-r} = \frac{108\sqrt{27}}{1-\frac{1}{\sqrt{27}}}$

$a = 108\sqrt{27}$

$\frac{108\sqrt{27}}{0.807\ldots}$

$S_\infty = \cancel{453.18\ldots}$

$= \cancel{453\ (3sf)}$

$S_\infty = 694.93\ldots$

$= 695\ (3sf)$

Question 15

Candidate 9

15. $\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 6y = 12x^2 + 2x - 5$

$M^2 + 5m + 6$

$(m+3)(m+2)$

$y = Ae^{2x} + Be^{3x} - 12x^2 - 2x + 5$

when $y = -6$ $\frac{dy}{dx} = 3$ $x = 0$

$\frac{dy}{dx} = 2Ae^{2x} + 3Be^{3x} - 24x - 2$

$-6 = A + B + 5$

$3 = 2A + 3B - 2$

$5 = 2A + 3B$

$-11 = A + B$

$27 = B$

$-11 = A + B$

$-11 = A + 27$

$-16 = A$

$y = -16e^{2x} + 27e^{3x} - 12x^2 - 2x + 5$