Q14 (a)

Candidate 8 evidence

14) $L_1 : x = 4 + 3\lambda,\ y = 2 + 4\lambda,\ z = -7\lambda$

$L_2 : x = -2s + 3,\ y = s + 8,\ z = 3s - 1$

Compare x's: $4 + 3\lambda = -2s + 3$

$3\lambda = -2s - 1$

$\lambda = -\frac{2}{3}s - \frac{1}{3}$ ①

Compare y's: $2 + 4\lambda = s + 8$

$4\lambda = s + 6$

$\lambda = \frac{1}{4}s + \frac{3}{2}$ ②

Compare z's: $-7\lambda = 3s - 1$

$\lambda = -\frac{3}{7}s + \frac{1}{7}$ ③

From ① & ②: $-\frac{2}{3}s - \frac{1}{3} = \frac{1}{4}s + \frac{3}{2}$

$\frac{11}{12}s = -\frac{11}{6}$

$s = -2$

$\Rightarrow \lambda = 1$

Check in ③ $\frac{11}{12}s = -\frac{11}{6}$

$\frac{11}{12}(-2) = -\frac{11}{6}$

$-\frac{11}{6} = -\frac{11}{6}$ ✓ $\Rightarrow$ The two lines intersect

Point ~~of~~ intersection $(7, 6, -7)$

Candidate 9

14a) $L_1 : \quad x = 4 + 3\lambda \quad y = 2 + 4\lambda \quad z = -7\lambda$

$L_2 : \frac{x-3}{-2} = \frac{y-8}{1} = \frac{z+1}{3} = t$

$x = 3 - 2t \quad y = t + 8 \quad z = 3t - 1$

$x : \; 3 - 2t = 4 + 3\lambda \qquad 3\lambda + 2t = -1 \quad (1)$

$y : \; t + 8 = 2 + 4\lambda \qquad 4\lambda - t = 6 \quad (2)$

$z : \; 3t - 1 = -7\lambda \qquad 7\lambda + 3t = 1 \quad (3)$

$3\lambda + 2t = -1 \quad (1)$

$8\lambda - 2t = 12 \quad (4)$

$(1) + (4)$

$11\lambda = 11$

$\underline{\lambda = 1}$

sub in (2)

~~8λ + 2t t=4~~

$4\lambda - t = 6$

$4 - t = 6$

$-t = 2$

$\underline{t = -2}$

check (3)

$7 - 6 = 1$ ✓

$x = 4 + 3(1)$

$\underline{= 7}$

$y = 2 + 4(1)$

$\underline{= 6}$

$z = -7\lambda$

$\underline{= -7}$

point of intersection $(7, 6, -7)$

Q10

Candidate 10

10. A. If p is prime then so is $2p+1$

$p = 2$ $\quad 2p+1$
$= 4+1$
$= 5$

$p = 5$ $\quad 2p+1$
$= 10+1$
$= 11$

$p = 7$ $\quad 2p+1$
$= 14+1$
$= 15$ $\quad$ 15 is not a prime number

therefore the statement A is false

B. $4 \div 3$
$= 1 \text{ r } 1$

$(4)^3 = 64 \div 3$
$= \cancel{8}21 \text{ r } 1$

$16 \div 3$
$= \cancel{4}5 \text{ r } 1$

$(16)^3 = 4096$
$= 1365 \text{ r } 1$

$22 \div 3$
$= 7 \text{ r } 1$

$(22)^3 = 10648$
$= 3549 \text{ r } 1$

The examples above show that Statement B is true

Candidate 11

ENTER NUMBER OF QUESTION

10. A. p is prime

let $p = 7$ 7 is prime

so $2(7)+1 = 15$, 15 is not prime so

proved by counter example that statement

A is false.

B. $n = 3m+1$

$n^2 = (3m+1)^2$

$= 9m^2 + 6m + 1$

$n^3 = (3m+1)(9m^2+6m+1)$

$= (27m^3 + 18m^2 + 3m + 9m^2 + 6m + 1)$

$= 3(9m^3 + 6m^2 + m + 3m^2 + 2m) + 1$

$= 3m+1$

so by direct proof statement B is true.

Candidate 12

NUMBER OF QUESTION

10) ~~A true.~~ A False

~~$p \neq a$, a is divisible by a and 1 only $a \in \mathbb{Z}$~~

~~$2p = 2a$~~

~~$2p+1 = 2a+1$~~

~~Assume if p is prime => 2p+1 is not prime~~

~~Counter example - if p is not prime => 2p+1 is prime~~

~~let $2p+1 = a$, a is only divisible by a & 1, $a \in \mathbb{Z}$~~

~~$2p = a-1$~~

~~$p = \frac{a}{2} - \frac{1}{2}$ ← Any prime clr~~

B ~~False counter example~~

True

Assume n has r1 when divided by 3 the n^3 does not have r1 when divided by 3

let $n = 3j + 1$

$$\Rightarrow n^3 = (3j+1)(3j+1)(3j+1)$$

$$= (9j^2 + 6j + 1)(3j+1)$$

$$= 27j^3 + 9j^2 + 18j^2 + 6j + 3j + 1$$

$$= 27j^3 + 27j^2 + 9j + 1$$

$$= 3(9j^3 + 9j^2 + 3j) + 1$$

$$= 3a + 1, \quad a \in \mathbb{Z}$$

Assumption proven to be false as when n has r1 when divided by 3, n^3 has r1 when divided by 3 so original statement is true

10A) False

let $2p+1 = a$, a is only divisible by itself & 1

$p = \frac{a}{2} - \frac{1}{2}$ - any prime divided by 2 equals an even number $+ \frac{1}{2}$, even number are not prime as $2 \mid$ even number

$p = j + \frac{1}{2} - \frac{1}{2}$

$j \in \mathbb{Z}$, j is even

$p = j$

$\Rightarrow p$ is even $\Rightarrow p$ is not prime.

As counter example is proven to be true original statement must be false.

Q5

Candidate 13

5) ~~Prove when n=1~~

Prove true for $n=1$

$LHS = \sum_{r=1}^{1} r(3r-1)$ $\quad RHS = n^2(n+1)$

$= 1(3(1)-1)$ $\quad = (1)^2(1+1)$

$= 1(2)$ $\quad = 1(2)$

$= 2$ $\quad = 2$

Since LHS = RHS for $n=1$, Statement is true for $n=1$

Assume true for $n=k$

$\sum_{r=1}^{k} r(3r-1) = k^2(k+1)$

Required to prove true for $n=k+1$

$LHS = \sum_{r=1}^{k+1} r(3r-1)$

$= \sum_{r=1}^{k} r(3r-1) + (k+1)(3(k+1)-1)$

$= k^2(k+1) + (k+1)(3k+3-1)$

$= k^3 + k^2 + 3k^2 + 3k - k + 3k + 3 - 1$

$= k^3 + 4k + 5k + 2$

$= k^3 + 2k^2 + 2k^2 + 4k + k + 2$

$= (k^2+2k+1)(k+2)$

$= (k+1)^2(k+1+1)$

$= RHS$ $\quad \Rightarrow$ Statement is true for $n=k+1$

Aim:

$RHS = (k+1)^2(k+1+1)$

$= (k^2+2k+1)(k+2)$

$= k^3 + 2k^2 + 2k^2 + 4k + k + 2$

$= k^3 + 4k^2 + 5k + 2$

5 con.) Since statement is true for $n=1$, true for $n=k \Rightarrow$ true for $n=k+1$, then by the principle of mathematical induction, Statement is true $\forall n \in \mathbb{N}$

Candidate 14

5. $\sum_{r=1}^{n} r(3r-1) = n^2(n+1)$

When $n=1$

$\sum_{r=1}^{1} r(3r-1) = 1 \times 2 = 2$

$n^2(n+1) = 1 \times 2 = 2$ $\therefore$ LHS = RHS

$\Rightarrow$ true for $n=1$

Assume $n=k$

$\sum_{r=1}^{k} r(3r-1) = k^2(k+1)$

Proof $n=k+1$

$LHS = \sum_{r=1}^{k+1} r(3r-1) = \sum_{r=1}^{k} r(3r-1) + \sum_{r=k}^{k+1} r(3r-1)$

$= k^2(k+1) + (k+1)(3k+3-1)$

$= k^2(k+1) + (k+1)(3k+2)$

$= k^2(k+1) + (3k^2+2k+3k+2)$

$= k^3 + k^2 + 3k^2 + 5k + 2 = k^3 + 4k^2 + 5k + 2$

$RHS = (k+1)^2(k+2)$

$= (k^2+2k+1)(k+2)$

$= k^3 + 2k^2 + 2k^2 + 4k + k + 2$

$= k^3 + 4k^2 + 5k + 2$

$\therefore$ LHS = RHS

$\Rightarrow$ True for $n = k+1$

Since true for $n=1$, $n=k$, $n=k+1$, so it true for all N.

Candidate 15

5\. for $n = 1$

LHS $\sum_{r=1}^{1} 1(3(1)-1) = 2$ RHS $1^2(1+1)$

$= 2$ so true.

assume true for $n=k$.

$\sum_{r=1}^{k} r(3r-1) = k^2(k+1)$

conjecture for $n = k+1$

$\sum_{r=1}^{k+1} r(3r-1) = (k+1)^2(k+1+1) \longrightarrow (k^2+2k+1)(k+2)$

$= k^3 + 4k^2 + 5k + 2$

$= k^2(k+1) + (k+1)(3(k+1)-1)$

$= k^3 + k^2 + (k+1)(3k+2)$

$= k^3 + k^2 + 3k^2 + 2k + 3k + 2$

$= k^3 + 4k^2 + 5k + 2$

$= (k+2)(k^2+2k+1) =$ conjecture.

proved true for $n = 1$, assumed true for $n = k$ and proved true for $n = k+1$. therefore proved by induction that $\sum_{r=1}^{n} r(3r-1) = n^2(n+1)$ for all

$\forall n \in N$.