Question 5(a)

Candidate 1

5.(a)

$$\left(3x - \frac{2}{x^2}\right)^8$$

$8-3r$

$$= \binom{8}{r}(3x)^{8-r}\left(\frac{-2}{x^2}\right)^r$$

$$= \binom{8}{r}(3)^{8-r}(x)^{8-r}(-2)^r\,(-2)^r(x^{-2})^r$$

$$= \binom{8}{r}3^{8-r}x^{8-r}-2^r\;x^{-2r}$$

$$= \underline{\underline{\binom{8}{r}(3)^{8-r}(x)^{8-3r}(-2)^r}}$$

Candidate 2

QUESTION NUMBER

5.(a)

$$\left(3x-\frac{2}{x^2}\right)^8 \quad \binom{n}{r}x^{n-r}y^r = \binom{8}{r}(3x)^{8-r}\left(-\frac{2}{x^2}\right)^r$$

$$= \underline{\binom{8}{r}(3^{8-r})(x^{8-r})(-2^{\cancel{r}})^r(x^{-2r})}$$

Candidate 3

5.(a)

$$\left(3x - \frac{2}{x^2}\right)^8 = \left(3x + (-2x^{-2})\right)^8$$

$$\binom{n}{r} a^{n-r} b^r$$

$$= \binom{8}{r} 3x^{8-r} (-2x^{-2})^r$$

$$= \binom{8}{r} 3^{8-r} x^{8-r} (-2)^r x^{-2r}$$

$$= \binom{8}{r} 3^{8-r} (-2)^r x^{8-3r}$$

Candidate 4

QUESTION NUMBER

5.(a)

$$\left(3x - \frac{2}{x^2}\right)^8$$

$$\binom{8}{r}(3x)^{8-r} \cdot (-2x^{-2})^r$$

$$\binom{8}{r} 3^{8-r} \cdot x^{8-r} \cdot -2^r \cdot x^{-2r}$$

$$\binom{8}{r} 3^{8-r} \cdot -2^r \cdot x^{8-3r}$$

Question 7(a)

Candidate 5

7.(a) $\frac{dy}{dx} - 2y = 6e^{5x}$ $\quad x=0 \quad y=-1$

$\frac{dy}{dx} + P(x)y = Q(x)$ $\quad P(x) = -2 \quad Q(x) = 6e^{5x}$

$I(x) = e^{\int P(x)dx} = e^{\int -2dx} = e^{-2x}$

$I(x)y = \int I(x)Q(x)dx$

$e^{-2x}y = \int 6e^{5x}e^{-2x}dx$

$e^{-2x}y = \int \frac{6e^{5x}}{e^{2x}}dx$

$e^{-2x}y = \int 6e^{3x}dx$

$\frac{y}{e^{2x}} = \frac{6}{3}e^{3x} + C$

$\frac{y}{e^{2x}} = 2e^{3x} + C$

$y = 2e^{5x} + e^{2x}C$

$-1 = 2 + C$

$C = -3$

$y = 2e^{5x} - 3e^{2x}$

Candidate 6

QUESTION NUMBER	
7.(a)	

$$\frac{dy}{dx} - 2y = 6e^{5x} \qquad y=1 \quad x=0$$

$P(x) = -2$

$Q(x) = 6e^{5x}$

$I(x) = e^{\int p(x)dx}$

$= e^{\int -2\, dx}$

$= e^{-2x}$

$I(x)y = \int Q(x)I(x)$

$e^{-2x}y = \int 6e^{5x} \cdot e^{-2x}$

$e^{-2x}y = \int 6e^{3x}$

$e^{-2x}y = 2e^{3x} + c$

$y = \dfrac{2e^{3x}}{e^{-2x}} + c \Rightarrow$

$y = 2e^{3x} \cdot e^{2x} + c$

$y = 2e^{5x} + c$

$1 = 2 + c$

$c = -1$

$y = 2e^{5x} - 1$

Candidate 7

QUESTION NUMBER

7.(a)

$$\int \frac{dy}{dx} - 2y = \int 6e^{5x}$$

$$y - 2y^2 = 6/5 e^{5x} + C$$

$$-1 - 2(-1)^2 = 6/5 e^{5(0)} + C \quad , \text{when } x=0, y=-1$$

$$-3 = 6/5 + C$$

$$C = -21/5$$

$$\Rightarrow \underline{\underline{y - 2y^2 = 6/5 e^{5x} - 21/5}}$$

Question 10

Candidate 8

10.

$$y = x^{5x^2}$$

$$\ln y = \ln x^{5x^2}$$

$$\Rightarrow \ln y = 5x^2 \cdot \ln x$$

$$\frac{1}{y}\frac{dy}{dx} = 10x \ln x + 5x$$

$$\frac{dy}{dx} = (10x \ln x + 5x)\, x^{5x^2}$$

$u = 5x^2 \quad v = \ln x$

$u' = 10x \quad v' = \frac{1}{x}$

$u'v + uv'$

$10x \ln x + 5x^2 \times \frac{1}{x}$

Candidate 9

10.

$$y = x^{5x^2}$$

$$\frac{dy}{dx}\cdot\frac{1}{y} = 5x^2 \ln x$$

$$\frac{dy}{dx}\cdot\frac{1}{y} = 10x \ln x + \frac{5x^2}{x}$$

$$\frac{dy}{dx}\cdot\frac{1}{y} = 10x \ln|x| + 5x$$

$$\frac{dy}{dx} = y\left(10x \ln x + 5x\right)$$

$$\frac{dy}{dx} = x^{5x^2}\left(10x \ln x + 5x\right)$$

Candidate 10

10.

~~$y = x^{5^{x^2}}$~~ $\quad y = x^{10}$

~~$\ln y = x^2 \ln x^5$~~

~~$\frac{1}{y}\frac{dy}{dx} = \frac{x^2}{x^5} \cdot 5x^4$~~

~~$\frac{1}{y}\frac{dy}{dx} = \frac{5x^6}{x^5}$~~

~~$\frac{1}{y}\frac{dy}{dx} = 5x$~~

~~$\frac{dy}{dx} = y(5x)$~~

~~$\frac{dy}{dx} = x^{5^{x^2}}$~~

$y = x^{5x^2}$

$\ln y = 5x^2 \ln x$ $\quad u = 5x^2 \quad u' = 10x$

$v = \ln x \quad v' = \frac{1}{x}$

$\frac{1}{y}\frac{dy}{dx} = 10x\ln x + 5x$

$\frac{dy}{dx} = y(10x\ln x + 5x)$

$\frac{dy}{dx} = x^{5x^2}(10x\ln x + 5x)$

$\frac{dy}{dx} = 10x^{5x^3}\ln x + 5x^{5x^3}$

Question 11(b)

Candidate 11

11.(b)

$\frac{dV}{dt} = 10 \ell s^{-1}$ $\quad \cancel{\frac{dV}{dh} = \frac{dV}{dt}}$ $\quad \frac{dh}{dt} = ?$

$\cancel{\frac{dh}{dh} = \frac{dV}{dt} \times \frac{dt}{dh}}$ $\quad \frac{dh}{dt} = \frac{dv}{dt} \cdot \frac{dh}{dV}$

$\frac{dV}{dh} = \frac{u'v - uv'}{v2}$ $\quad u = 3\pi h^3$ $\quad v = 25$

$u' = 3\pi h^2 . 3$ $\quad v' = 0$

$= 9\pi h^2$

$= \frac{9.\pi h^2 . 25 - 3\pi h^3 . 0}{25^2}$

$\frac{dV}{dh} = \frac{225\pi h^2}{625} = \frac{9\pi h^2}{25}$ $\quad \frac{dh}{dt} = 10 . \frac{25}{9\pi h^2}$

$\frac{dh}{dV} = \frac{25}{9\pi h^2}$ $\quad \frac{dh}{dt} = \frac{250}{9\pi h^2}$

when $h = 125$ $\quad \frac{dh}{dt} = \frac{250}{9 \times \pi \times 125^2}$

$\frac{dh}{dt} = 5.659 \times 10^{-4}$

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Candidate 12

11.(b)

$\frac{dV}{dt} = 10$ $\qquad V = \frac{3\pi h^3}{25}$

$\frac{dV}{dh} = \frac{9\pi h^2}{25}$

$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV}$

$= 10 \times \frac{25}{9\pi h^2}$

$= \frac{250}{9\pi h^2}$

$= \frac{250}{9\pi (125)^2}$ when, $h = 125$

$= 5.6588... \times 10^{-4}$

$= 5.66 \times 10^{-4}$ cms to 3

Question 12

Candidate 13

QUESTION NUMBER

12.

$\sum_{r=1}^{n} 2^{r-1} r = 2^n(n-1)+1$

LHS

Step 1: $n=1$ $\sum_{r=1}^{1} 2^{r-1} r = 2^{1-1} \times 1 = 2^0 = 1$

RHS

$2^1(1-1)+1 = 2(0)+1 = 1$ LHS = RHS $\therefore$ True for $n=1$.

Step 2: Assume true for $n=k$

$\sum_{r=1}^{k} 2^{r-1} r = 2^k(k-1)+1$

Step 3: $n=k+1$

LHS $\sum_{r=1}^{k+1} 2^{r-1} r = \sum_{r=1}^{k} 2^{r-1} r + (2^{k+1-1} \times (k+1))$

$= 2^k(k-1)+1+2^k(k+1)$

$= \cancel{2^k((k-1)+1+(k+1))}$

$= \cancel{2^k(k-1+1+k+1)}$

$= \cancel{2^k}(2k+1)$

$= 2^k(k-1+k+1)+1$

$= 2^k(2k)+1$

RHS $2^{k+1}(k+1-1)+1$

$2^{k+1}(k)+1$

$2^k \times 2(k)+1$ $2^k \times 2k+1$

$\cancel{2^k(2k+1)}$ $2^k(2k)+1$

LHS = RHS $\therefore$ True for $n=k+1$

Step 4: Since true for $n=k+1$ when true for $n=k$ and also true for $n=1$, by induction the statement is true $\forall n \in \mathbb{N}$.

Candidate 14

12\. let $n=1$

LHS RHS

$\sum_{r=1}^{1} 2^{1-1} \times 1$ $\quad 2^1(1-1)+1$

$2(0)+1$

$= \sum_{r=1}^{1} 2^{1-1} \times 1$ $\quad =1$

$= 2^0 \times 1$ Since LHS = RHS, true for $n=1$

$= 1$

$2\sum 1^{r-1} \times \sum r$

Assume true for $n=k$

$$\sum_{r=1}^{k} 2^{r-1} r = 2^k(k-1)+1$$

consider $n=k+1$ target:

$\sum_{r=1}^{k+1} 2^{r-1} r$ = first term + k term $\quad \sum_{r=1}^{k+1} 2^{k+1}((k+1)-1)+1$

$$\sum_{r=1}^{k+1} 2^{r-1} r = 2^k(k-1)+1+2^{(k+1)-1}(k+1)$$

$$= 2^k(k-1)+1+2^k(k+1)$$

$$= 2^k((k-1)+(k+1))+1$$

$$= 2^k(2k+-1)+1$$

$$= 2^k.2((k+1)-1)+1$$

$$= 2^{k+1}((k+1)-1)+1$$

if it is true for $n=k$, then it is also true for $n=k+1$. But since it is true for $n=1$, then by induction it is true for all positivive integers n.

Candidate 15

QUESTION NUMBER	
12.	

Conjecture

$$\sum_{r=1}^{n} 2^{r-1} r = 2^{n}(n-1)+1$$

Consider $n=1$

$LHS = \sum_{r=1}^{1} 2^{r-1} r$

$= 2^{1-1}(1)$

$= 2^{0}(1)$

$= 1(1)$

$= 1$

$RHS = 2^{1}(1-1)+1$

$= 2(0)+1$

$= 0+1$

$= 1$

LHS=RHS true for $n=1$

Assume true for $n=k$

i.e. $\sum_{r=1}^{k} 2^{r-1} r = 2^{k}(k-1)+1$

consider $n=k+1$

Aim to prove: $\sum_{r=1}^{k+1} 2^{r-1} r = 2^{k+1}((k+1)-1)+1$

$LHS = \sum_{r=1}^{k+1} 2^{r-1} r$

$= \sum_{r=1}^{k} 2^{r-1} r + 2^{k+1-1}(k+1)$

$= 2^{k}(k-1)+1 + 2^{k}(k+1)$ by assumption

$= 2^{k}(k-1) + 2^{k}(k+1) + 1$

$= 2^{k}(k) - \cancel{2^{k}} + 2^{k}(k) + \cancel{2^{k}} + 1$

$= 2^{k}(k) \times 2 + 1$

$= 2 \cdot 2^{k}(k)+1$

$= 2^{k+1}(k)+1$

$= 2^{k+1}((k+1)-1)+1 = RHS$ as required

if $n=k$ true $\Rightarrow$ $n=k+1$ true also $n=1$ true $\therefore$ by induction, conjecture true $\forall n \in \mathbb{Z}^{+}$

Question 13

Candidate 16

13.

$$(m-220)\frac{dp}{dm} = 1.4p$$

$$\int \frac{1}{(m-220)}\,dm = \int \frac{1}{1.4p}\,dp$$

$$\ln|m-220| = \frac{1}{1.4}\ln|p| + c$$

$m = 807 \quad p = 1079$

$$\ln|807-220| = \frac{1}{1.4}\ln|1079| + c$$

$$6.37502\ldots = \frac{1}{1.4}\times 6.9837\ldots + c$$

$$6.38 = 4.99 + c$$

$$6.38 - 4.99 = c$$

$$c = 1.39$$

$$\ln|m-220| = \frac{1}{1.4}\ln|p| + 1.39$$

~~m-220~~

$$\ln|m-220| - 1.39 = \frac{1}{1.4}\ln|p|$$

$$1.4(\ln|m-220| - 1.39) = \ln|p|$$

$$p = e^{1.4(\ln|m-220| - 1.39)}$$