Question 3

Candidate 1

ENTER NUMBER OF QUESTION

3a) $\left(2x + \frac{5}{x^2}\right)^9$ $\quad (a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r$

$\sum_{r=0}^{9} \binom{9}{r} (2x)^{9-r} \left(\frac{5}{x^2}\right)^r$

$\sum_{r=0}^{9} \binom{9}{r} (2)^{9-r} (5)^r (x)^{9-r} (x)^{-2r}$

$\sum_{r=0}^{9} \binom{9}{r} (2)^{9-r} (5)^r (x)^{9-3r}$

b) x^{9-3r} $\quad 9-3r=0$

$-3r=-9$

$r=3$

$\sum_{r=0}^{9} \binom{9}{3} (2)^{9-3} (5)^3 x^0$

$\frac{9!}{3!\,6!} (2)^{9-3} (5)^3$

$= 84 \times 64 \times 125$

$= 672000$

Candidate 2

3a $\binom{9}{n}(2x)^{9-n}\left(\frac{5}{x^2}\right)^n = \binom{9}{n}2^{9-n}x^{9-n} \cdot \frac{5^n}{x^{2n}}$

$= \binom{9}{n}\frac{2^{9-n}\times 5^n \times x^{9-n}}{x^{2n}}$

b. $9-n-2n=0$

$9-3n=0$

$9=3n$

$n=3$

$\binom{9}{3}\frac{2^6\times 5^3\times x^6}{x^6}$

$= 84\times 2^6\times 5^3$

$= 672000$

Candidate 3

ENTER NUMBER OF QUESTION

3 $\sum_{r=0}^{9} \cancel{\left(2x + \frac{5}{x^2}\right)} (2x)^{9-r}\left(\frac{5}{x^2}\right)^r \, {}^9C_r$

$= {}^9C_0(2x)^9 + {}^9C_1(2x)^8\left(\frac{5}{x^2}\right) + {}^9C_2(2x)^7\left(\frac{5}{x^2}\right)^2$

$+ {}^9C_3(2x)^6\left(\frac{5}{x^2}\right)^3 + {}^9C_4(2x)^5\left(\frac{5}{x^2}\right)^4 + {}^9C_5(2x)^4\left(\frac{5}{x^2}\right)^5$

$+ {}^9C_6(2x)^3\left(\frac{5}{x^2}\right)^6 + {}^9C_7(2x)^2\left(\frac{5}{x^2}\right)^7 + {}^9C_8(2x)\left(\frac{5}{x^2}\right)^8$

$+ {}^9C_9\left(\frac{5}{x^2}\right)^9$

$\sum_{r=0}^{9} {}^9C_r(2x)^{9-r}\left(\frac{5}{x^2}\right)^r = 512x^9 + 11520x^6 +$

$115200x^3 + 672000 +$

$\frac{2520000}{x^3} + \frac{6300000}{x^6}$

$+ \frac{1050000}{x^9} + \frac{11250000}{x^{12}}$

$+ \frac{7031250}{x^{15}} + \frac{1953125}{x^{18}}$

ENTER NUMBER OF QUESTION

b) $\sum_{r=0}^{9} {}^{9}C_r (2x)^{9-r} \left(\frac{5}{x^2}\right)^r$

$= \sum_{r=0}^{9} {}^{9}C_r \frac{2^{9-r} x^{9-r} 5^r}{x^{2r}}$

$\frac{x^{9-r}}{x^{2r}} = x^{9-3r}$

$\therefore$ term ind. of x at x^0

$9-3r=0$

$9=3r$

$\underline{\underline{r=3}}$

when $r=3$

${}^{9}C_3 (2x)^6 \left(\frac{5}{x^2}\right)^3$

$84 \times 64 \times 125 = 672,000$

Question 9

Candidate 4

9) $x + (x+1) + (x+2) = 3k$

$x + x + 1 + x + 2 = 3k$

$3x + 3 = 3k$

$3(x+1) = 3k$

where $\underline{\underline{K = x+1}}$

b) $2k+1 = x + x + 1$

$= 2x + 1$

$\underline{\underline{\text{where } x = k}}$

Candidate 5

4. a) let n be an integer

$n-1$, n and $n+1$ are three consecutive integers

$(n-1)+n+(n+1)$

$= 3n$ this is divisible by 3

$\therefore$ the sum of any three integers is divisible by 3

4. b) let n be an integer

n and $n+1$ are consecutive

$n + n+1 = 2n+1$

this is the form of an odd integer $\therefore$ an odd integer can be expressed as the sum of two consecutive integers

Candidate 6

9a.

$n+(n+1)+(n+2)$

$= n+n+1+n+2$

$= 3n+3$

$= 3(n+1)$

$3 \mid 3(n+1)$

multiple of 3 $\therefore$ the sum is divisible by 3

9b.

$2n+1$ ← $2n+1$ is odd as $2n$ is even $+1$ is odd

$= n+(n+1)$ ← $n+1$ is the consecutive integer of n

Candidate 7

9 a $(2k-1)+(2k)+(2k+1) = 6k = 3(2k)$ $\therefore \Sigma$ any 3 consecutive integers is devisible by 3

b $(2k)+(2k+1) = 4k+1 = 2(2k)+1$

Question 12

Candidate 8

12) $\sum_{r=1}^{n} 3^{r-1} = \frac{1}{2}(3^n - 1)$

let $n=1$

left hand side (LHS) | Right hand side (RHS)
$3^{1-1} = 3^0 = 1$ | $\frac{1}{2}(3^1-1)$
 | $\frac{1}{2}(2)$
 | $= 1$

LHS = RHS
so true for $n=1$

assume true for $n=k$

$\sum_{r=1}^{k} 3^{r-1} = \frac{1}{2}(3^k - 1)$

consider it to be true for $n=k+1$

$\sum_{r=1}^{k+1} 3^{r-1} = \frac{1}{2}(3^{(k+1)} - 1)$ <— Aim

$\sum_{r=1}^{k+1} 3^{r-1} = \sum_{r=1}^{k} 3^{r-1} + 3^{(k+1)-1}$

$= \frac{1}{2}(3^k - 1) + 3^{(k+1)-1}$

$= \frac{1}{2}(3^k - 1) + 3^{(k-1)} \times 3$

$= \frac{1}{2}(3^k - 1 + 6^{k-1} \times 6)$

$= \frac{1}{2}(3^k - 1 + (2 \times 3^{k-1}) \times 3 \times 2)$

Candidate 9

ENTER NUMBER OF QUESTION

12 $\sum_{r=1}^{n} 3^{r-1} = \frac{1}{2}(3^n - 1)$

when $n=1$

3^{1-1} $\quad \frac{1}{2}(3^1-1)$

$= 3^0$ $\quad = \frac{1}{2}(2)$

$= 1$ $\quad = 1$

RHS = LHS $\therefore$ statement true for $n=1$

assume also true for $n=k$

$\sum_{r=1}^{k} 3^{r-1} = \frac{1}{2}(3^k - 1)$

when $n=k+1$

$\sum_{r=1}^{k+1} 3^{r-1} = \sum_{r=1}^{k} 3^{r-1} + 3^{((k+1)-1)}$

$= \frac{1}{2}(3^k-1) + 3^{((k+1)-1)}$

~~$= \frac{1}{2}3k - \frac{1}{2} + 3$~~

$= \frac{1}{2}3^k - \frac{1}{2} + 3^k \quad = \frac{1}{2}3^k + 3^k \quad \frac{\cdot}{2} \quad - \frac{1}{2}$

$= (3^k-1) + 2.3^{((k+1)-1)}$ $\quad \frac{1}{2}(9^k - 1$

$3^{(k+1)-1}$

$= 3^k - 1 + 2.3^{((k+1)-1)}$

$= \frac{3^k}{3^{(k+1)-1}} - \frac{1}{3^{(k+1)-1}} + 2$

$1 - \frac{1}{3^{(k+1)-1}} + 2$

$= \frac{1}{2}(3^k - 1) + \frac{3^{k+1}}{3^1}$

$= \frac{3}{2}(3^k - 1) + 3^{k+1} = \frac{1}{2}(3(3^k-1)$

aim

$\frac{1}{2}(3^{k+1}-1)$

Candidate 10

12) $\sum_{r=1}^{n} 3^{r-1} = \frac{1}{2}(3^n - 1)$

for $n=1$

$$\sum_{r=1}^{1} 3^{r-1} = \frac{1}{2}(3^1 - 1)$$

$$3^0 = \frac{1}{2}(2)$$

$$\underline{\underline{1 = 1}}$$

assume for $n=k$

$$\sum_{r=1}^{k} 3^{r-1} = \frac{1}{2}(3^k - 1)$$

prove for $n=k+1$

$$\sum_{r=1}^{k+1} 3^{r-1} = \frac{1}{2}(3^{k+1} - 1)$$

$$\sum_{r=1}^{k} 3^{r-1} + 3^{k+1-1} = \frac{1}{2}(3^{k+1} - 1)$$

Candidate 10

DO NOT WRITE IN THIS MARGIN

LHS

$$\sum_{r=1}^{k} 3^{r-1} = \frac{1}{2}(3^{k} - 1)$$

$$\frac{1}{2}(3^{k} - 1) + 3^{k}$$

$$= \frac{3^{k}}{2} - \frac{1}{2} + 3^{k}$$

$$= \frac{3^{k}}{2} - \frac{1}{2} + \frac{2 \times 3^{k}}{2}$$

$$= \frac{3 \times 3^{k}}{2} - \frac{1}{2}$$

$$\frac{3^{1} \times 3^{k}}{2} - \frac{1}{2}$$

$$= \frac{3^{k+1}}{2} - \frac{1}{2} = \text{RHS}$$

$$\frac{1}{2}(3^{k+1} - 1) = \frac{3^{k+1}}{2} - \frac{1}{2}$$

Proved for $n=1$
assumed for $n=k$ $\therefore$ Proven by induction.
Proved for $n=k+1$

Question 15 (b)

Candidate 11

b) $\frac{dy}{dx} - \frac{2}{x}y = x^3 \sin 3x$

$P(x) = -\frac{2}{x}$ $\quad Q(x) = x^3 \sin 3x$

$I(x) = e^{\int -\frac{2}{x} dx}$

$= e^{-2\ln x}$

$= e^{\ln x^{-2}}$

$= \frac{1}{x^2}$

$I(x)y = \int I(x)Q(x)$

$\frac{1}{x^2} y = \int \frac{1}{x^2} \times x^3 \sin 3x$

$\frac{1}{x^2} y = \int x \sin 3x$

$\frac{1}{x^2} y = -\frac{1}{3} x \cos 3x + \frac{1}{9} \sin 3x + C$

$y = -\frac{1}{3} x^3 \cos 3x + \frac{1}{9} x^2 \sin 3x + x^2 C$

$y = \frac{x^2}{3}\left(-x \cos 3x + \frac{1}{3} \sin 3x + C\right)$

Candidate 12

(b)

$$\frac{dy}{dx} - \frac{2y}{x} = x^3 \sin 3x$$

$$\text{I.F.} = e^{\int P(x)\,dx}$$

$$= e^{\int -\frac{2}{x}}$$

$$= e^{-2\ln x}$$

$$= e^{\ln x^{-2}}$$

$$= x^{-2} = \frac{1}{x^2}$$

$$\frac{d}{dx}\left(\frac{y}{x^2}\right) = x \sin 3x$$

$$\frac{y}{x^2} = \frac{-x\cos 3x}{3} + \frac{\sin 3x}{9} + C$$

$$y = -\frac{x^3\cos 3x}{3} + \frac{x^2 \sin 3x}{9} + C$$

Candidate 13

b $\frac{dy}{dx} - \frac{2}{x}y = x^3 \sin 3x$

$u = x^3$ $\quad \frac{dv}{dx} = \sin 3x$

$\frac{du}{dx} = 3x^2$ $\quad v = \frac{-1}{3}\cos 3x$

$P(x) = \frac{-2}{x}$ $\quad Q(x) = x^3 \sin 3x$

$I(x) = e^{\int P(x)dx}$

$= e^{\int -2/x\,dx}$

$= e^{-2\ln x}$

$= x^{-2}$

$= \frac{1}{x^2}$

$\cos 3\pi = -1$

$I(x)y = \int Q(x)\,dx$

$\frac{1}{x^2}y = \int x^3 \sin 3x\,dx$

$\frac{1}{x^2}y = \int uv - \int v\frac{du}{dx}$

$= \frac{-x^3}{3}\cos 3x + x^2\cos 3x + c$

$y = \frac{-x^5}{3}\cos 3x + x^4 \cos 3x + c$

When $y = 0$ $x = \pi$

$0 = -\frac{(\pi)^5}{3}\cos(3\pi) + (\pi)^4\cos(3\pi) + c$

$c = 4.6$

$f(x) = -x^5/3 \cos 3x + x^4 \cos 3x + 4.6$

Question 16 (d)

Candidate 14

d) $\pi_2 = \begin{pmatrix} 3 \\ -5 \\ -2 \end{pmatrix}$ $\pi_4 = \begin{pmatrix} -9 \\ 15 \\ 6 \end{pmatrix} = -3\begin{pmatrix} 3 \\ -5 \\ -2 \end{pmatrix}$

They lie on the same plane

$\underline{\underline{\pi_2 = \pi_4}}$

Candidate 15

d) they are parralled

there direction vectors are a multiple of each other

Candidate 16

(d) ~~Since the [illegible] been swapped~~

Since the coefficients of x, y, and z have been multiplied by -3, the ~~[illegible]~~ points on the plane are -3 times larger comparatively